FACTORING SHORTCUTS 12.1.1

Although most factoring problems can be done with generic rectangles, there are two special factoring patterns that, if recognized, can be done by sight. The two patterns are known as **Difference of Squares** and **Perfect Square Trinomials.** The general patterns are as follows:

Difference of Squares: $a^2x^2 - b^2 = (ax+b)(ax-b)$

Perfect Square Trinomial: $a^2x^2 + 2abx + b^2 = (ax+b)^2$

Examples

Difference of Squares	**Perfect Square Trinomials**
$x^2 - 49 = (x+7)(x-7)$	$x^2 - 10x + 25 = (x-5)^2$
$4x^2 - 25 = (2x-5)(2x+5)$	$9x^2 + 12x + 4 = (3x+2)^2$
$x^2 - 36 = (x+6)(x-6)$	$x^2 - 6x + 9 = (x-3)^2$
$9x^2 - 1 = (3x-1)(3x+1)$	$4x^2 + 20x + 25 = (2x+5)^2$

Sometimes removing a common factor reveals one of the special patterns.

Example 1

$8x^2 - 50y^2 \Rightarrow 2(4x^2 - 25y^2) \Rightarrow 2(2x+5y)(2x-5y)$

Example 2

$12x^2 + 12x + 3 \Rightarrow 3(4x^2 + 4x + 1) \Rightarrow 3(2x+1)^2$

Problems

Factor each Difference of Squares.

1.	$x^2 - 16$	2.	$x^2 - 25$	3.	$64m^2 - 25$
4.	$4p^2 - 9q^2$	5.	$9x^2y^2 - 49$	6.	$x^4 - 25$
7.	$64 - y^2$	8.	$144 - 25p^2$	9.	$9x^4 - 4y^2$

Factor each Perfect Square Trinomial.

10.	$x^2 + 4x + 4$	11.	$y^2 + 8y + 16$	12.	$m^2 - 10m + 25$
13.	$x^2 - 4x + 16$	14.	$a^2 + 8ab + 16b^2$	15.	$36x^2 + 12x + 1$
16.	$25x^2 - 30xy + 9y^2$	17.	$9x^2y^2 - 6xy + 1$	18.	$49x^2 + 1 + 14x$

Factor completely.

19.	$9x^2 - 16$	20.	$9x^2 + 24x + 16$	21.	$9x^2 - 36$
22.	$2x^2 + 8xy + 8y^2$	23.	$x^2y + 10xy + 25y$	24.	$8x^2 - 72$
25.	$4x^3 - 9x$	26.	$4x^2 - 8x + 4$	27.	$2x^2 + 8$

Answers

1.	$(x+4)(x-4)$	2.	$(x+5)(x-5)$	3.	$(8m+5)(8m-5)$
4.	$(2p+3q)(2p-3q)$	5.	$(3xy+7)(3xy-7)$	6.	$(x^2+5)(x^2-5)$
7.	$(8+y)(8-y)$	8.	$(12+5p)(12-5p)$	9.	$(3x^2+2y)(3x^2-2y)$
10.	$(x+2)^2$	11.	$(y+4)^2$	12.	$(m-5)^2$
13.	not possible	14.	$(a+4b)^2$	15.	$(6x+1)^2$
16.	$(5x-3y)^2$	17.	$(3xy-1)^2$	18.	$(7x+1)^2$
19.	$(3x+4)(3x-4)$	20.	$(3x+4)^2$	21.	$9(x+2)(x-2)$
22.	$2(x+2y)^2$	23.	$y(x+5)^2$	24.	$8(x+3)(x-3)$
25.	$x(2x+3)(2x-3)$	26.	$4(x-1)^2$	27.	$2(x^2+4)$

ADDITION AND SUBTRACTION OF RATIONAL EXPRESSIONS 12.1.2 and 12.1.3

Addition and Subtraction of Rational Expressions uses the same process as simple numerical fractions. First, find a common denominator (if necessary). Second, convert the original fractions to equivalent ones with the common denominator. Third, add (or subtract) the new numerators over the common denominator. Finally, factor the numerator and denominator and reduce (if possible). See the Math Notes box on page 503. Note that these steps are only valid provided that the denominator is not zero.

Example 1

The least common multiple of $2(n+2)$ and $n(n+2)$ is $2n(n+2)$.

$$\frac{3}{2(n+2)}+\frac{3}{n(n+2)}$$

To get a common denominator in the first fraction, multiply the fraction by $\frac{n}{n}$, a form of the number 1. Multiply the second fraction by $\frac{2}{2}$.

$$=\frac{3}{2(n+2)}\cdot\frac{n}{n}+\frac{3}{n(n+2)}\cdot\frac{2}{2}$$

Multiply the numerator and denominator of each term. It may be necessary to distribute the numerator.

$$=\frac{3n}{2n(n+2)}+\frac{6}{2n(n+2)}$$

Add, factor, and simplify the result. (Note: $n \neq 0$ *or* 2)

$$=\frac{3n+6}{2n(n+2)}\Rightarrow\frac{3(n+2)}{2n(n+2)}\Rightarrow\frac{3}{2n}$$

Example 2

$$\frac{2-x}{x+4}+\frac{3x+6}{x+4}\quad\Rightarrow\quad\frac{2-x+3x+6}{x+4}\quad\Rightarrow\quad\frac{2x+8}{x+4}\quad\Rightarrow\quad\frac{2(x+4)}{x+4}\quad\Rightarrow\quad 2$$

Example 3

$$\frac{3}{x-1}-\frac{2}{x-2}\quad\Rightarrow\quad\frac{3}{x-1}\cdot\frac{x-2}{x-2}-\frac{2}{x-2}\cdot\frac{x-1}{x-1}\quad\Rightarrow\quad\frac{3x-6-2x+2}{(x-1)(x-2)}\quad\Rightarrow\quad\frac{x-4}{(x-1)(x-2)}$$

Problems

Add or subtract each expression and simplify the result. In each case assume the denominator does not equal zero.

1. $\dfrac{x}{(x+2)(x+3)}+\dfrac{2}{(x+2)(x+3)}$

2. $\dfrac{x}{x^2+6x+8}+\dfrac{4}{x^2+6x+8}$

3. $\dfrac{b^2}{b^2+2b-3}+\dfrac{-9}{b^2+2b-3}$

4. $\dfrac{2a}{a^2+2a+1}+\dfrac{2}{a^2+2a+1}$

5. $\dfrac{x+10}{x+2}+\dfrac{x-6}{x+2}$

6. $\dfrac{a+2b}{a+b}+\dfrac{2a+b}{a+b}$

7. $\dfrac{3x-4}{3x+3}-\dfrac{2x-5}{3x+3}$

8. $\dfrac{3x}{4x-12}-\dfrac{9}{4x-12}$

9. $\dfrac{6a}{5a^2+a}-\dfrac{a-1}{5a^2+a}$

10. $\dfrac{x^2+3x-5}{10}-\dfrac{x^2-2x+10}{10}$

11. $\dfrac{6}{x(x+3)}+\dfrac{2x}{x(x+3)}$

12. $\dfrac{5}{x-7}+\dfrac{3}{4(x-7)}$

13. $\dfrac{5x+6}{x^2}-\dfrac{5}{x}$

14. $\dfrac{2}{x+4}-\dfrac{x-4}{x^2-16}$

15. $\dfrac{10a}{a^2+6a}-\dfrac{3}{3a+18}$

16. $\dfrac{3x}{2x^2-8x}+\dfrac{2}{x-4}$

17. $\dfrac{5x+9}{x^2-2x-3}+\dfrac{6}{x^2-7x+12}$

18 $\dfrac{x+4}{x^2-3x-28}-\dfrac{x-5}{x^2+2x-35}$

19. $\dfrac{3x+1}{x^2-16}-\dfrac{3x+5}{x^2+8x+16}$

20. $\dfrac{7x-1}{x^2-2x-3}-\dfrac{6x}{x^2-x-2}$

Answers

1. $\dfrac{1}{x+3}$

2. $\dfrac{1}{x+2}$

3. $\dfrac{b-3}{b-1}$

4. $\dfrac{2}{a+1}$

5. 2

6. 3

7. $\dfrac{1}{3}$

8. $\dfrac{3}{4}$

9. $\dfrac{1}{a}$

10. $\dfrac{x-3}{2}$

11. $\dfrac{2}{x}$

12. $\dfrac{23}{4(x-7)}=\dfrac{23}{4x-28}$

13. $\dfrac{6}{x^2}$

14. $\dfrac{1}{x+4}$

15. $\dfrac{9}{a+6}$

16. $\dfrac{7}{2(x-4)}=\dfrac{7}{2x-8}$

17. $\dfrac{5(x+2)}{(x-4)(x+1)}=\dfrac{5x+10}{x^2-3x-4}$

18. $\dfrac{14}{(x-7)(x+7)}=\dfrac{14}{x^2-49}$

19. $\dfrac{4(5x+6)}{(x-4)(x+4)^2}$

20. $\dfrac{x+2}{(x-3)(x-2)}=\dfrac{x+2}{x^2-5x+6}$

Work problems are solved using the concept that if a job can be completed in r units of time, then its rate (or fraction of the job completed) is $\frac{1}{r}$.

Mixture problems are solved using the concept that the product (value or percentage) x (quantity) must be consistent throughout the equation.

Examples of Work Problems

John can completely wash and dry the dishes in 20 minutes. His brother can do it in 30 minutes. How long will it take them working together?

Solution: Let t = the time to complete the task so $\frac{1}{t}$ is their rate together. John's rate is $\frac{1}{20}$ and his brother's rate is $\frac{1}{30}$. Since they are working together we add the two rates together to get the combined rate.

The equation is $\frac{1}{20}+\frac{1}{30}=\frac{1}{t}$. Solve using fraction busters:

$$60t\left(\tfrac{1}{20}+\tfrac{1}{30}\right)=60t\left(\tfrac{1}{t}\right)$$
$$3t+2t=60$$
$$5t=60$$
$$t=12 \text{ minutes}$$

With two inflowing pipes open, a water tank can be filled in 5 hours. If the larger pipe can fill the tank alone in 7 hours, how long would the smaller pipe take to fill the tank?

Solution: Let t = the time for the smaller pipe so $\frac{1}{t}$ is its rate. The combined rate is $\frac{1}{5}$ and the larger pipe's rate is $\frac{1}{7}$. Since they are working together we add the two rates together to get the combined rate.

The equation is $\frac{1}{7}+\frac{1}{t}=\frac{1}{5}$. Solve using fraction busters:

$$35t\left(\tfrac{1}{7}+\tfrac{1}{t}\right)=35t\left(\tfrac{1}{5}\right)$$
$$5t+35=7t$$
$$35=2t$$
$$t=17.5 \text{ hours}$$

Examples of Mixture Problems

Alicia has 10 liters of an 80% acid solution. How many liters of water should she add to form a 30% acid solution?

Solution: Use $(\%)\,(liters) = (\%)\,(liters)$

Let x = liters of water added so $x + 10$ is new liters.

The equation is

$$(0.8)\,(10) = (0.3)\,(x+10).$$

Multiply by 10 to clear decimals and solve.

$$(8)(10) = (3)(x+10)$$

$$80 = 3x + 30$$

$$50 = 3x$$

$$x = 16\tfrac{2}{3}\ liters$$

A store has candy worth \$0.90 a pound and candy worth \$1.20 a pound. If the owners want 60 pounds of candy worth \$1.00 a pound, how many pounds of each candy should they use?

Solution: Use $(\$)\,(lbs.) = (\$)\,(lbs.)$

Let x = pounds of \$0.90 candy

so $60 - x$ = pounds of \$1.20 candy.

The equation is

$$0.90(x) + 1.20(60 - x) = 1.00(60).$$

Multiply by 100 to clear decimals and solve.

$$90(x) + 120(60 - x) = 100(60)$$

$$90x + 7200 - 120x = 6000$$

$$-30x = -1200$$

$$x = 40$$

40 lbs. of \$0.90 candy and
20 lbs. of \$1.20 candy

> Note: The second example above could also have been solved using two equations, where $x = \$0.90$ candy and $y = \$1.20$ candy:
>
> $$x + y = 60 \ \ and \ \ \$0.90x + \$1.20y = \$60.00$$

Problems

Solve each problem.

1. Susan can paint her living room in 2 hours. Her friend Jaime estimates it would take him 3 hours to paint the same room. If they work together, how long will it take them to paint Susan's living room?

2. Professor Minh can complete a set of experiments in 4 hours. Her assistant can do it in 6 hours. How long will it take them to complete the experiments working together?

3. With one hose a swimming pool can be filled in 12 hours. Another hose can fill it in 16 hours. How long will it take to fill the pool using both hoses?

4. Together, two machines can harvest a tomato crop in 6 hours. The larger machine can do it alone in 10 hours. How long does it take the smaller machine to harvest the crop working alone?

5. Steven can look up 20 words in a dictionary in an hour. His teammate Mary Lou can look up 30 words per hour. Working together, how long will it take them to look up 100 words?

6. A water tank is filled by one pump in 6 hours and is emptied by another pump in 12 hours. If both pumps are operating, how long will it take to fill the tank?

7. Two crews can service the space shuttle in 12 days. The faster crew can service the shuttle in 20 days alone. How long would the slower crew need to service the shuttle working alone?

8. Janelle and her assistant Ryan can carpet a house in 8 hours. If Janelle could complete the job alone in 12 hours, how long would it take Ryan to carpet the house working alone?

9. Able can harvest a strawberry crop in 4 days. Barney can do it in 5 days. Charlie would take 6 days. If they all work together, how long will it take them to complete the harvest?

10. How much coffee costing \$6 a pound should be mixed with 3 pounds of coffee costing \$4 a pound to create a mixture costing \$4.75 a pound?

11. Sam's favorite recipe for fruit punch requires 12% apple juice. How much pure apple juice should he add to 2 gallons of punch that already contains 8% apple juice to meet his standards?

12. Jane has 60 liters of 70% acid solution. How many liters of water must be added to form a solution that is 40% acid?

13. How many pound of nuts worth \$1.05 a pound must be mixed with nuts worth \$0.85 a pound to get a mixture of 200 pounds of nuts worth \$0.90 a pound?

14. A coffee shop mixes Kona coffee worth \$8 per pound with Brazilian coffee worth \$5 per pound. If 30 pounds of the mixture is to be sold for \$7 per pound, how many pounds of each coffee should be used?

15. How much tea costing \$8 per pound should be mixed with 2 pounds of tea costing \$5 per pound to get a mixture costing \$6 per pound?

16. How many liters of water must evaporate from 50 liters of an 8% salt solution to make a 25% salt solution?

17. How many gallons of pure lemon juice should be mixed with 4 gallons of 25% lemon juice to achieve a mixture which contains 40% lemon juice?

18. Brian has 20 ounces of a 15% alcohol solution. How many ounces of a 50% alcohol solution must he add to make a 25% alcohol solution?

Answers

1.	1.2 hours	2.	2.4 hours	3.	$6\frac{6}{7} \approx 6.86$ hours
4.	15 hours	5.	2 hours	6.	12 hours
7.	30 days	8.	24 hours	9.	$\frac{60}{37} \approx 1.62$ days
10.	1.8 pounds	11.	$\frac{1}{11}$ gallon	12.	45 liters
13.	50 pounds	14.	20 Kona, 10 Brazilian	15.	1 pound
16.	34 liters	17.	1 gallon	18.	8 ounces