Latinos and Mathematics

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INTRODUCTION

The subject of Latinos and mathematics is an exciting and intriguing one because questions of mathematics and culture are intimately intertwined. A historical perspective on this matter reveals that sophisticated mathematical activity has been going on in the Latino culture for thousands of years.

Mathematics Education and Ethnomathematics

Mathematics education dates its beginnings to the time when human beings began to quantify the objects and phenomena in their lives. Although the process of counting was the same for different groups of people around the world, the symbols by which they represented specific quantities varied according to their own particular cultural conventions. Thus, the Babylonians, Romans, Hindus, Egyptians and other African cultural groups, Chinese, Mayas, Aztecs, Incas, and others wrote numbers differently.

Those cultures that achieved a level of mathematical sophistication allowing them to manipulate their number symbols (to add, subtract, multiply, divide, and perform other algorithms) performed those manipulations in different ways. Today, even within a single society, various groups of people (e.g. accountants, physicists, engineers, mathematicians, chemists) view and manipulate mathematical quantities differently. The study of the way that specific cultural "ethno" groups go about the tasks of classifying, ordering, counting, measuring, and otherwise mathematizing their environment is called *ethnomathematics* (D'Ambrosio 1985 a, p. 2; 1985 b, p. 44).

Latinos and Mathematics Education

The population group in the United States referred to as Hispanics include mostly people who can trace their origins to Latin America. The largest group is of Mexican, the next largest of Puerto Rican, and the third largest of Cuban descent. The rest of our Hispanic population is of Central and South American origin. Although people of Latin America are aware that their language (Spanish), their names, and their dominant religion (Catholicism) are cultural traits transplanted from Spain to the New World, they prefer to call themselves Latino rather than Hispanic. Hence, the term Latino in the title of this essay.

Latinos are the fastest growing population group in the United States. Current demographic projections forecast that they will become the largest minority group in the year 2013 (Orange County Register, 1992). Today, the median age of this group is about twenty, while the median age of the general population is thirty-three. The U.S. Census Bureau estimates this latter median will rise to thirty-five in the year 2035 while the median age of Latinos will rise to the low twenties. Thus, it is in the the national interest that we improve the achievement of Latinos in mathematics and science to prepare them to replace the white scientists and technicians who will leave the working force in the next several decades.

This Baseline Essay concerns the ethnomathematics topic of Latinos and mathematics. Many Latinos trace their roots to countries like México, Guatemala, El Salvador, and Perú, where the modern national cultures are heavily influenced by both indigenous and Spanish cultural traditions. This essay concerns pre-Columbian mathematics in Latin America and the contributions of Latinos to modern mathematics. We approach the subject from the historical perspective of the mathematical accomplishments of three pre-Columbian civilizations (Olmecs, Aztecs, and Incas), mathematical activities in Latin America during the Spanish Colonial period, and brief sketches of Latinos in modern mathematics in the United States. The intention is to illustrate the fact that mathematics has been an important part of the cultural activity of Latinos and that this ethnic group can exhibit high achievement in the subject.

Those mathematics teachers who are aware of this cultural tradition have used it in their classroom to motivate Latino students to do well in mathematics. For example, in a scene in the movie *Stand and Deliver*, teacher Jaime Escalante attempts to impel his Latino students to excel in mathematics by saying, "You burros have math in your blood." As we shall see in this essay, Latinos do indeed have mathematics in their cultural blood.

It is vitally important that ethnomathematics topics like those in this essay be taught in North American schools, because such educational practices benefit both school children and society at large. The United States of America has experienced profound demographic changes in the last fifteen years that have significantly altered its cultural landscape.

Current population projections indicate that the percentage of school-age children who are African-American, Latino, Asian, and Native American will grow faster in the years to come than the percentage of white children. As a result, the demand for multiculturalism in education will increase.

In the view of the National Association of State Boards of Education (NASBE), a multicultural learning environment is one in which multicultural perspectives are integrated into academic topics throughout the curriculum rather than being taught as "special" topics in the schools. The NASBE (c. 1992, p. 1) has observed, "Our ability to move schools and institutions into the next century rests on our ability to remove the obstacles that hinder multicultural learning environments." One of those obstacles is the absence of multicultural perspectives in mathematics.

When children are exposed to ethnomathematics topics such as the one discussed in this essay, their appreciation for the achievement of other cultures in mathematics will increase. Moreover, the perception of students that only western culture has created mathematical knowledge will be changed. Thus, ethnomathematics is a vehicle for providing our nation's students with a fuller and richer educational experience.

PRE-COLUMBIAN MATHEMATICS

When Columbus accidentally arrived in the New World on October 12, 1492, the two dominant cultures in these new-found lands were the Aztecs of México and the Incas of Perú. However, before 1492 a long tradition of astonishing intellectual and cultural accomplishments had already been established by other civilizations in what is now known as Mesoamerica (the geographical region that encompasses the area from north-central México to northern Costa Rica in Central America) and the region in South America that includes parts of Colombia, Ecuador, Perú, Bolivia, Chile, and Argentina.

The cultural activity in Mesoamerica was characterized by a successive evolution of civilizations that benefited from the advances and accomplishments of preceding cultures through a process of cultural transmission that is largely unknown to us. From archaeological data, it appears that the Olmec is the oldest civilization of Mesoamerica and the Aztec is the most recent. However, there were other civilizations that rose, flourished, and declined in Mesoamerica between the Olmecs and the Aztecs. Some of these civilizations were in some respects more advanced than the Olmec and Aztec, but their names (Mayas, Zapotecs, etc.) will be mentioned only in connection with the mathematics topic being discussed.

Mesoamerica was the cradle of two vigesimal (base twenty) number systems. One included the use of zero as a place holder, and the other did not include the use of zero. The most sophisticated indigenous civilization in South America that we know of is the Inca civilization of Perú, which created a decimal (base ten) number system.

The mathematical activity in the cultural life of the civilizations that used these three number systems included architecture (construction of homes and temples/pyramids), astronomy, commerce, construction of roads, and record keeping. While record keeping and commerce only required the use of "basic" mathematics, architecture, astronomy, and road construction required the use of geometry and engineering. In the pages that follow, we will set out a description of the three number systems created by the Olmecs, Aztecs, and Incas and refer to some of their applications. We will discuss the two Mesoamerican vigesimal systems first and then the decimal system of the Incas.

Vigesimal System of the Olmecs

A numerical inscription using bars and dots dating before 1200 B.C. (Soustelle 1984, p. 60) was found in an archaeological site of the Olmecs. This date precedes the rise of the classic Maya civilization by more than 1,300 years. In fact, the Olmec—the oldest known civilization of Mesoamerica—is credited with creating both the number system and the calendar system that is commonly attributed to the Mayas (see, for example, Caso 1965, p. 932; Schele and Freidel 1990, pp. 79-80; Soustelle 1984, pp. 25, 140, and Chapter 9; and Stuart and Stuart 1983, pp. 38).

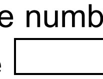
The number system created by the Olmecs was a positional vigesimal system, base twenty, employing only three symbols to write any whole number from zero to whatever quantity was desired. The three symbols are the figure , which represents zero; the dot, • , which represents the quantity of one; and the horizontal bar, —, which represents the quantity of five. To write the numbers zero to nineteen in this system, the two processes of grouping and addition are used (see Figure 1). Five dots are grouped into a bar and the numbers 2-4 and 6-19 are written using the addition process. Numbers larger than 19 are written vertically following a positional convention.

Figure 1. Olmec Numbers 0 to 19

In this convention, the bottom level is for the units, the next level up is for the 20s, the third level up is for the 400s (1 x 20 x 20), the fourth level up is for the 8,000s (1 x 20 x 20 x 20), and so on in powers of 20. For instance, to write 20, we write the figure in the first level and a dot, •, in the second level. To write 65, we write ••• in the second level, 3 x 20 = 60, and a —, for five units, in the first level. These two quantities and some additional ones are illustrated in Figure 2.

Figure 2. Examples of numbers beyond 19 in Olmec notation:
(a) 20; (b) 65; (c) 805; (d) 10,951 and (e) 40,329

This vigesimal number system was used by many cultures of Mesoamerica in various everyday applications. Those cultures that used it in their calendrics to record dates of events employed a modified version of the system. In that version, a unit in the third level had the value of 360 rather than 400 (see Figure 3). This was the result of multiplying the units in the second level by 18 to get to the third level, 1 x 20 x 18. The value of the units beyond the third level followed the vigesimal convention. For instance, units in the fourth level had the value of 7,200 derived from 1 x 20 x 18 x 20, units in the fifth level had the value of 144,000 derived from 1 x 20 x 18 x 20 x 20, and so on for subsequent levels. For other illustrations on the use of this calendar system, see Krause (1983, pp. 45-46). Although this number system is called Mayan, it was used by cultures predating the Mayas (e.g. Olmec, Zapotec, and others. See section below, "Origins of the Olmec Vigesimal System".)

Figure 3. Two examples of chronological counts in the Olmec system

This vigesimal system of numeration is very practical to use and can be easily adapted to classroom instruction. The operations of addition and subtraction are relatively straightforward processes. In the case of addition, one simply has to remember that since twenty units in a lower level are equivalent to one unit in the next level up, twenty units in a lower level are replaced with one dot in the level above it (the "regrouping process"). Figure 4 shows the sum of 8,095 plus 1,166, illustrating the regrouping process.

Figure 4. The sum of 8,095 plus 1,166. Column (b) shows the result 9,261 before regrouping and (c) is the final result, after regrouping

In the case of subtraction, when borrowing is required, one unit from a higher level equals twenty units in the next lower level. The subtraction process of 40,329-10,951 is illustrated in Figure 5. Notice that in order to perform this subtraction, some borrowing is necessary. One unit is borrowed from the 8,000s level in the number on the left, which then becomes 20 units in the 400s level, and one unit is borrowed from the 20s level and transferred to the units level as 20. Figure 5 (b) shows the modified representation of 40,329. Figure (c) illustrates the result of the subtraction 40,329-10,951.

Figure 5. The subtraction 40,329-10,951 showing the intermediate step (b) with borrowing process in 40,329 and the result 29,378 in (c)

Unfortunately, we do not know whether the Mesoamerican civilizations knew how to multiply, divide, or carry out other mathematical algorithms with this vigesimal system. But we do know that the Mayas wrote books on paper just as we do. We know that for 1,600 years before Columbus accidentally arrived in the New World, the Mayas wrote and had kept thousands of books in which they recorded their history and cultural achievements (Schele and Freidel 1990, pp. 18 and 401). Tragically, the Spanish conquerors and missionaries burned and otherwise destroyed all of the Mayan libraries and archives. It is possible that some of those books contained information on the algorithms in the mathematical systems as done by the pre-Columbian societies. We do know that Mayan astronomers calculated the cycles of the heavens so precisely that they could predict solar and lunar eclipses to the day hundred of years before they happened. In fact, it was a Mayan astronomer who first predicted, some 1,200 years in advance, the solar eclipse that occurred on July 11, 1991 (*Newsweek*, July 15, 1991, pp. 58-62).

The Mayas knew the synodical revolution of Venus (Thompson 1966, p. 170; Morley, Brainerd, and Sharer 1983, p. 566), and some scholars argue that the Mayas also knew the synodical period of Mars and perhaps had similar knowledge about Mercury, Jupiter, and Saturn (Morley, Brainerd, and Sharer 1983, p. 567). A synodical period or revolution refers to the time it takes for two heavenly bodies, the earth and Venus for example, to pass from one conjunction to another. To make these calculations as well as to predict celestial phenomena, it is reasonable to believe, the Mayas knew how to perform mathematical algorithms other than addition and subtraction. This belief is rooted in the origins and uses of their vigesimal number system.

Origins of the Olmec Vigesimal Number System

The Olmecs are known to us for the huge human heads they sculpted on stone. When people see pictures of them in books or actual examples in museum exhibits, they observe one Negroid facial feature and speculate about possible African influences in Mesoamerican civilizations. Although the debate about the origins of humans in the New World has not been settled yet, there is neither anthropological nor archaeological evidence to support conjectures about possible African influences.

Archaeological data support the notion that the Olmecs achieved their cultural accomplishments independently from other civilizations. As far as we know, no other civilization in the world invented a numerical system like the one described in the preceding pages. Furthermore, no other civilization, prior to or contemporaneous with the Olmecs, used this number system to create a calendar system to record dates as the Olmecs did.

Based on these facts, the arguments that attempt to take credit from the Olmecs for their cultural accomplishments are dubious. Moreover, such arguments sound subtly demeaning because of the implication that Mesoamerican civilizations were not capable of such intellectual achievements.

Archaeologists and scholars maintain that humans first inhabited North America around 30,000 years ago and México, in particular, about 9,000 years later (Lorenzo 1977, p.102). Groups of hunters and gatherers roamed Mesoamerica for thousands of years before they reached the sedentary stage. Soustelle (1984, p. 4) pins down the advent of agriculture in Mesoamerica at around 4000 B.C. However, the organized life that can be called civilization in the region began approximately about 5,000 years ago (Schele and Freidel 1990, p. 37). The social evolution of Mesoamerica can be traced from the hunter-gatherers through the successive civilizations of the Olmecs, Zapotec, Mayas, Toltecs, Aztecs, etc.

The earliest evidence of numerical inscriptions using the positional systems of bars and dots has been traced to the Olmecs in approximately 1200 B.C. (Soustelle 1984, p. 60). The historical significance of this date is that it preceded by some 800 years the accomplishments of Aristotle, Plato, and Euclid, whose western culture did not yet have a positional number system. It was not until 499 A.D. that the Hindu-Arabic number notation using zero in a positional convention first occurred (Ganguli 1932, p.251). The Zapotecs of Oaxaca used the Olmec notation in their calendrics between 900 and 400 B.C. (Soustelle 1984, pp. 102-104). Between 400 B.C. and 300 A.D., the Izapan culture used this same convention (Stuart and Stuart 1983, pp. 25-26). Later, the Mayas, to whom the vigesimal system is mistakenly attributed, used this system extensively between A.D. 199 and 900. The Mayas developed their amazingly complex calendar system and astronomical sciences around this mathematical system hundreds of years before Galileo and Copernicus lived.

The above historical evidence is compelling data to postulate the hypothesis that the Mayas knew how to perform mathematical algorithms other than just addition and subtraction.

Recommendations to Teachers

The Olmec positional number system discussed in this essay can be taught at various educational levels. It can be included in the elementary school curriculum as a way to sharpen the students' understanding of the decimal system. In fact, the Mesoamerican system may be easier for children to grasp than the decimal system for at least two reasons. One, the vigesimal system is very visual; and two, the representation of the quantities involved (zero, one, and five) can be easily adapted to concrete representation with manipulative materials.

For example, Dienes blocks can be adapted to the Mesoamerican base twenty number system. The teacher would assign the value of one to the smallest blocks, the value of five to intermediate-sized blocks, and the value of twenty to larger blocks or to a group of four intermediate-sized blocks. Alternatively, in those classrooms where Dienes blocks are not available but where manipulative materials of different colors and sizes (rods or chips) are used for arithmetics activities, different colors or sizes of rods or chips may be used for the three quantitative designations of one, five, and twenty.

When doing addition in this vigesimal system, teachers and students must remember the rule: "One should not have more than twenty." Whenever there are more than twenty units in one cell, the student must make a twenty-for-one replacement and place the replacement in the cell above. In the case of subtraction when borrowing is involved, one unit in a higher level is replaced with twenty units in the next level down. See Figures 4 and 5 for examples of how to do addition and subtraction in this vigesimal system.

At the middle school and high school levels, discussion of this number system can be included in social studies courses as well as in mathematics classes in order to broaden the students' appreciation of the cultural achievements of ancient peoples and the fate of conquered civilizations. Teachers can use this topic to illustrate that the impressive intellectual achievements of Mesoamerican civilizations were ignored, devalued, or destroyed as part of the rationale for subjugation and domination. For presentations in social studies courses at these levels, a map of Mesoamerica is indispensable and can be obtained from the National Geographic Society or from some of the references listed at the end of this essay. (See for example Stuart and Stuart 1983, pp. 21 and 51; and Stuart 1981, pp. 98-99). Teachers are encouraged to consult these references for more details on the historical origins and uses of this numerical system and for more information on the pre-Columbian cultures that used it (see for example Morley, Brainerd, and Shaker 1983; Ortiz-Franco and Magaña 1973; and Thompson 1966).

In mathematics classes at the middle school and high school levels, teachers can devise exercises relating powers of ten to the value of digits in numerals in the decimal number system and the value of units in the vigesimal system. The pattern to be observed is this: while the value of a digit in the decimal system is multiplied by a power of ten that corresponds to the place of the digit of the numeral, the value of the same number of units in the same corresponding place in the vigesimal system is multiplied by a power of twenty. This can lead to discussions about powers of twenty as a product of powers of two times powers of ten to illustrate the fact that the value of units in the vigesimal system increases exponentially faster than the value of these units in the corresponding place in the decimal system. This in turn can serve as a natural introduction to a discussion of topics related to exponential growth and exponential functions.

Furthermore, in mathematics classes where students are already proficient with the multiplication algorithm in the decimal system (grades 5-12), teachers can include classroom activities or homework assignments requiring students to use their creativity when working with the vigesimal system. For instance, the teacher may break up the class into groups of three or four students each and ask the groups to generate ideas on how to carry out multiplication in this number system. This idea can be extended to include division as well. These challenging assignments may turn into group projects that can last for an extended period of time. The inclusion of such extended projects or investigations is one way of implementing aspects of the Curriculum and Evaluation Standards of the National Council of Teachers of Mathematics. More importantly, these extended projects address what the Curriculum and Evaluation Standards ignores: the importance of doing mathematics in a multicultural context.

Vigesimal System of the Aztecs

Like the Olmecs, the Aztecs employed a vigesimal number system (Cajori 1928a, p. 41; and Vaillant 1962, p. 171). But unlike the Olmecs, the Aztecs did not use a symbol for zero because theirs was not a positional number system. Moreover, the Aztecs used more than three symbols to write their numbers.

There are two conventions to represent the quantity of one in this vigesimal system. Both Cajori 1928a, and Vaillant 1962 agree that a dot, •, represents the number one. Vaillant, but not Cajori, says that a finger was also used to represent the quantity of one. To write numbers from one to nine, one simply has to repeat the dot or the finger as many times as required, e.g. two dots or fingers for the number two, five dots or fingers for number five, etc.

Cajori 1928a, p. 41, says that the Aztecs wrote the number ten like a lozenge, □, but Vaillant 1962, p. 171, does not mention this symbol at all. However, both Cajori and Vaillant indicate that ten dots were also used to write the number ten. For numbers up to nineteen, one could use any of the following four conventions: 1) the requisite number of dots; 2) the requisite number of fingers; 3) a combination of a lozenge and dots, or 4) a combination of a lozenge and finger for numbers from ten to nineteen.

The number twenty was represented by a flag, , and 400 was represented with a sign like a fir or a feather with twenty hairs or barbs—each of which represented twenty. One hundred was written by using either five flags or by drawing five barbs or hairs on the feather. Two hundred and three hundred were written by drawing ten barbs and fifteen barbs, respectively. A purse was used to represent the number 8,000. Notice that $400 = 20 \times 20 = 20^2$, and $8,000 = 20 \times 20 \times 20 = 20^3$. However, the symbols that the Aztecs might have used for numbers $160,000 = 20^4$ or $3,200,000 = 20^5$ and higher powers are unknown to us. The symbols for numbers, using dots instead of fingers for the units, in the Aztec number system are as shown in Figure 6, below.

Figure 6. Aztec numerals using dots for units instead of fingers

As we can see, the Aztec number system is considerably clumsier than the Olmec system. To write large numbers, or to add and subtract quantities, is very cumbersome due to all the symbols one must draw for the numbers involved. Moreover, because the Aztec number system does not use place value, one can write numbers beyond ten by arranging the respective symbols for the quantities involved in any order desired. For example, to write the number fifteen, one can either write a lozenge for ten first, followed by five dots for five, • • • • • or one can reverse the order of these symbols, • • • • •, and still convey the same quantity.

The multiple possibilities of writing numbers beyond ten in the Aztec number system are further demonstrated when we consider the alternate ways of representing the units, dots or fingers, and writing the number ten (a lozenge or ten dots or ten fingers). For instance, the number 8,375 can be written in more than 240 different ways, depending on which way we want to order the number symbols involved, and which symbol we want to use for the units and number ten. Figure 7 below illustrates only two ways in which we can write 8,375. Moreover, Figure 7 illustrates the additive principle employed when writing large numbers in the Aztec system.

Figure 7. Two alternate ways of writing 8,375 in Aztec numerals

Although it is possible to perform addition and subtraction using Aztec numerals, we will discuss those operations with small numbers here because these operations with large numbers can be very cumbersome and confusing. Moreover, because this number system does not use place value, the task of doing these two operations with Aztec numerals demands from us some adaptations in the way we are used to manipulating and viewing numbers.

The succeeding Figure 8, illustrates the sum of 37 and 45 with Aztec numerals using the convention of writing numbers of descending values from left to right and using a lozenge for ten and dots for units. To perform addition, we just add as many similar number figures as there are in both lines and write the resulting numeral. We then do as much regrouping as necessary to write the final result in the most compact form possible.

Figure 8. The sum of 37 and 45 with Aztec numerals:
(a) shows the result before regrouping; (b) illustrates the intermediate step of regrouping ten dots into a lozenge; and (c) shows the final form of 82 after regrouping two lozenges into a flag

Following the same convention in writing numerals as in Figure 8, the subtraction of 19 from 46 is illustrated in Figure 9, below. To perform subtraction, sometimes it is necessary to replace larger numerals with equivalent quantities using number figures of lesser value. For example, a flag representing twenty can be replaced with two lozenges and a lozenge can be replaced with ten dots, and so on for other replacements that might be needed. After these adjustments are made, subtract the number figures in the bottom line from similar number figures in the top line. In other words, we cancel an equal number of similar figures in the top and the bottom lines, and the remaining numeral is the desired result.

Figure 9. The subtraction of 19 from 53: (a) illustrates the two numbers, 53 and 19, and (b) shows the result, 34, after one lozenge is replaced with ten dots and one flag is replaced with two lozenges, and the appropriate cancellations are carried out

Origins of the Aztec Vigesimal System

The literature on the Aztec number system does not mention other cultures using a similar system. Hence, we cannot say whether the Aztec vigesimal number system was created by the Aztecs or they borrowed it from another culture.

The history of the Aztec civilization indicates that the Aztecs moved around from place to place until they established their home base in Tenochtitlan, now México City, in July 1325. The Aztec empire was conquered by the Spanish in 1521. Sometime during these intervening 200 years, the Aztecs developed their written language as we know it today. However, it is not clear precisely when during those two centuries the Aztecs began to write their numbers as described above.

However clumsy and cumbersome the Aztec number system is, the simple fact that the Aztecs represented their numerals in written symbolic form allowed them to manipulate those symbols in order to carry out mathematical calculations—quite an intellectual achievement.

Recommendations to Teachers

The variety of ways of writing numbers in the Aztec vigesimal system makes it impossible to suggest any single method to adapt this number system to the classroom. However, the paragraphs below outline some practical ideas on how to demonstrate some aspects of the Aztec vigesimal system in the classroom.

Teachers should adopt the convention of writing the units using dots and the number ten using the lozenge. Furthermore, the convention of writing numerals by writing the symbols for numbers in descending order from left to right is very practical. These three conventions will eliminate some of the confusion that may arise from using different symbols for the units, and the number ten, and may eliminate the confusion from mixing the order of the number symbols. We must realize, however, that even these conventions, if adopted, will not eliminate the cumbersome aspect of doing mathematics with Aztec numerals.

Alternatively, instead of teachers making these decisions they can seize the opportunity to promote creativity by allowing students to decide for themselves. These classroom activities may be done individually or in groups. A group activity of this type encourages two desirable educational goals: communication and collaboration in the mathematics classroom.

The ideas mentioned in the paragraph above illustrate one way of implementing three aspects in mathematics education advocated by the NCTM Curriculum and Evaluation Standards (the Standards): creativity, communication, and collaboration. However, this essay promotes these activities in a multicultural mathematics context not mentioned in the NCTM document.

Teachers in grades K-2 are in a good position to use Aztec numerals in doing addition and subtraction as long as the numbers involved are not too large. Even doing addition and subtraction with numbers larger than forty can be cumbersome. Teachers in grades 2-12 should limit their discussion of Aztec numerals to topics related to demonstrations of how different cultures have their own conventions of writing numbers. Activities or assignments at these latter grade levels may include asking students to generate as many ways as possible to write a numeral, say 675 or 8,375, using Aztec number symbols.

Decimal System of the Incas

As far as we know, the Inca civilization of Perú was the most advanced pre-Columbian culture in South America. Unlike the Olmecs and the Aztecs, neither the Incas nor those cultures that preceded them for over 2,000 years in South America had a writing system (Von Hagen, Ed., 1969, Footnote No. 2, pp. 39-40; McIntyre 1984, p. 30). They gave their numbers a physical representation in a system of knotted strings called *quipus*. The *quipu* was a twisted woolen cord or rope upon which other smaller cords of different colors were tied.

The color, length, and number of knots on the *quipus*, the distance of one knot from another, and turns within a knot, all had their significance (Cajori 1928b, p. 12; McIntyre 1984, p. 30). By these knots, the Incas counted from one to ten, from ten to one hundred, and from one hundred to one thousand (von Hagen 1969, p. 174), using a base-ten system (McIntyre 1984, p. 31; Zepp 1992, p. 42). In this decimal system a string with no knots signified zero, thus making it evident that the Incas understood the concept of zero (McIntyre 1984, p.31). For additional comments on the *quipus* and the possible ways of representing large numbers in a single string, please consult Ascher (1981 and 1992) and the essay "American Indian Mathematics Traditions and Contributions" in this Baseline series.

To represent numbers in the *quipu* system, the farthest string to the right is for the units, the next string to the left is for the tens, the next string to the left is for the hundreds, and so on, just like the place value of digits in our own decimal system. The number of knots on a given string indicates the number of units in that particular place value. Figure 10 below shows the representation of several numbers in the quipu system.

Figure 10. *Quipus* representing the numbers 302; 1,015; 30,184; and 15

McIntyre (1984, p. 31) says that *quipus* were not used to compute, and Zepp (1992, p. 42) cites sources stating that the Incas performed calculations on some sort of abacus. However, Cajori (1928b, p. 12-13) shows a picture of a *quipu* that illustrates the possibility of a sum of four numbers in a *quipu*. But, it is quite plausible that the sum of the four numbers that Cajori refers to might have been done using an abacus and the result recorded on a *quipu*, because he also says that the *quipus* were not adapted for computation (Cajori 1928b, p. 13). In any event, the Incas used the *quipus* as a mnemonic system to aid the memory (von Hagen, Ed.,1969, Footnote No. 2, pp. 39-40).

Quipus were used to keep account of provisions in storehouses, population census (von Hagen, Ed., 1969, pp. 105 and 163), tabulation of crop and livestock production (McIntyre 1984, p. 30), and other matters, such as events that took place in the kingdom. Different colors of strands were used to record different things. The number of knots in a string, the number of twists in the knots, and the colors of the strings conveyed information that could only be interpreted or remembered by specialized people called *quipu-camayocs* ("rememberers"). Figures 11A, 11B, and 11C, below, show examples of how complex the *quipu* system could be.

Figure 11A

Figure 11B

Figure 11C

Figures 11 A, B, C. Examples of elaborate *quipus* that served the Incas to record quantitative and historical data; Source: Loren McIntyre (1984). "The Incredible Incas and Their Timeless Land." *National Geographic*, page 30.

The Incas were so sophisticated in the use of the *quipus* that some scholars theorize they transcribed sounds of their language into numbers (Zepp 1992, p. 42). This is quite possible when we consider that according to von Hagen (1969, p. 187), *quipu-camayocs* could relate in a sequential order events that had taken place in the kingdom, as is done in ballads and lays. Moreover, McIntyre (1984, p. 89) says that many ordinances were recorded in *quipus*, such as time limits on the mourning for the dead, restrictions on women's activities in time of war, and conditions governing the performance of marriages. Furthermore, McIntyre (1984, p. 118) states that, during civil wars, the Incas customarily burned the *quipus* and killed the *quipu-camayocs* to erase the past.

The conquering Spaniards in 1532 and thereafter also burned rooms full of *quipus* to stamp out "pagan" practices, and they even seized personal *quipus* (McIntyre 1984, p. 31). The cultural value of the loss of so much historical data is incalculable. We will never know the mathematical calculations that the Incas might have performed with *quipus* and abacuses in the process of building their amazing aqueducts and road systems. These architectural and engineering accomplishments required the use of sophisticated indigenous mathematical techniques that took many years to develop. We also know that the Incas recorded their considerable astronomical knowledge in *quipus* (Cajori 1928b, p. 13). Even though thousands of ancient *quipus* are frequently unearthed by archaeologists in Inca ruins, without "remembers" to interpret these records, the strings are silent.

Origins of the Inca Decimal System

Because neither the Incas nor the South American cultures preceding them had a writing system, it is difficult to trace the cultural development and the systems of cultural transmission in Perú. This obstacle prevents us from pinning down an approximate time when the Incas or their predecessors began to mathematize their environment using *quipus*. However, modern archaeological techniques reveal that for some 4,500 years before the Incas, human culture was blossoming in Perú.

A brief summary of the history of pre-Columbian civilizations in Perú is offered by von Hagen 1961, pp. 408-418. The highlights are as follows: Archaeological evidence shows the presence of weaving and agriculture in Perú as early as 3000 B.C. and the presence of pottery around 1,500 years later. The first culture of prominence was Chavin at around 1200 B.C. (pp. 410-411). A wave of experimentation in weaving and pottery by many cultures in the region took place between 400 B.C. and A.D. 400. Subsequently, a period of high craftsmanship in architecture, ceramics, and weaving and the presence of cities occurred around A.D. 400-1000 (p. 411). Between A.D. 1000 and 1300, the empire of Tiahuanaco was the dominant civilization in Perú and Bolivia (pp. 414-415). The Chimu Empire, A.D. 1000-1466, ruled in Perú and Ecuador. It was the last of the larger cultures to offer opposition to the Incas (p. 417).

The Inca Empire dawned in A.D. 1438 and lasted until A.D. 1532 when Francisco Pizarro led a band of Spaniards to conquer this mighty culture. At their height, the Incas extended their dominion about 2,500 miles and left their stamp in Perú, Bolivia, Ecuador, Colombia, Chile, and Argentina (McIntyre 1984, p. 12). However, because none of the cultures that thrived for 4,500 years in this region had writing, we do not know when the Incas or their predecessors created the *quipu* system that included the concept of zero.

Recommendations to Teachers

Because the decimal number system represented in *quipus* works precisely as does our own decimal system, teachers can adapt the Inca system of numeration to any activity in mathematics classes that involves addition or subtraction. Admittedly, tying seven or nine knots on a string becomes tedious, but this can be overcome by replacing knots with beads to represent units on a string. Moreover, beads of different colors can stand for reference to different objects or people. Thus, a teacher can encourage students to create games with codes in the *quipu* system using beads of different colors.

Another idea for teachers is suggested by Zepp (1992, p. 44): assign the numbers from one to twenty-six to the letters of the alphabet (A=1; B=2, C=3, etc.) and have students write their names or messages to each other in *quipu* code. Although Zepp does not mention the use of beads, his idea can be easily implemented using beads of different colors to make the code games more exciting for children.

COMMENTARY ON PRE-COLUMBIAN MATHEMATICS

Because of space limitations for this essay, we have limited our discussion to three pre-Columbian civilizations that flourished in what is now Latin America. But consistent with the basic notions of ethnomathematics—that groups of people create counting or number systems to mathematize the phenomena around them and their cultural experiences—other pre-Columbian civilizations had their own mathematical conventions.

For instance, Harvey and Williams (1986, pp. 237-259) argue that the Texcocans, contemporaries of the Aztecs who also thrived in what is now México City, had a positional number system even though they did not have a special symbol for zero. They wrote their numbers differently from the Aztecs and the Olmecs. For additional articles on this general topic, the interested reader on pre-Columbian mathematics should consult Closs (1986).

From an historical perspective, the transition from oral forms of thought to the written or symbolic representation of concepts and ideas signifies a sophisticated level of both cognitive and cultural development. For many people, mathematical thought is one of the most abstract intellectual activities. In mathematical circles, number systems that do not include a symbol for zero, and its associated role in writing numerals following a place-value convention, are not as advanced as those number systems that do. From cognitive, cultural, and mathematical perspectives, the three pre-Columbian number systems described in this essay illustrate three different levels of mathematical thought that flourished in the New World prior to the conquest.

From a mathematical point of view, the Aztec number system is considered to be less sophisticated than the Olmec and Inca number systems because it did not include either a symbol for zero or the concept of place value. On the other hand, although the Inca decimal system did include a symbol for zero and place value, the Incas did not reach a level of cultural development that allowed them to use written symbols for their numerals, as did the Aztecs and the Olmecs. The Olmecs, however, reached both of these advanced levels of mathematical thought (symbolic representation of zero and use of place value in numerals) much earlier than either the Aztecs or the Incas.

The use of zero to represent numerals in a place-value convention is considered to be a watershed development in mathematical thought. Such an important accomplishment has occurred only three times in the history of civilization. The Olmecs of Mesoamerica have the distinction of being the first culture in the world to have achieved this intellectual landmark, at around 1200 B.C. The next time this mathematical achievement occurred was around 499 A.D. in India, and the third time was with the Inca culture sometime between 1438 and 1532 A.D. Thus, there is indeed a heritage of excellence in mathematics in the culture of Latinos.

The recommendations to teachers outlined in the preceding three sections are but a few of the many ways in which pre-Columbian mathematics can be integrated in the classroom.

Mathematics curriculum supervisors and teachers are encouraged to consult other sources to complement those recommendations. For example, Krause (1983) includes some games of chance and number activities pertaining to the tree cultures discussed herein. The essay on American Indian mathematics traditions in this Baseline series is another source.

MATHEMATICAL ACTIVITY IN LATIN AMERICA

The accidental arrival of Columbus in the New World on October 12, 1492, marked the beginning of a 300-year period of Spanish colonial rule in Latin America which ended as early as the 1820s in some countries. The military conquest of the indigenous civilizations by Spanish soldiers was followed by the imposition of Western culture on peoples who had developed their world view hundreds, and in some places thousands, of years before the Spanish arrived. In the realm of mathematics, this meant that the mathematical systems and practices of these cultures were supplanted, as part of the process of domination, by the Hindu-Arabic decimal system used by the Spanish.

México and Perú were the two focal points of Spanish colonial rule in Latin America. The first two European-style universities in the New World were established in these two countries in the middle of the sixteenth century: the University of México in México, and the University of San Marcos in Lima, Perú. Without seeing copies of the curriculum of these two universities, it is reasonable to expect that mathematics was taught. The following facts make it evident that there was some interest in mathematics in colonial Latin America.

The currents in mathematical activity in Spain during the sixteenth through the early part of the nineteenth century had an influence in Latin America due to Spanish colonial policies. There were two main lines of mathematical thought in sixteenth-century Spain: the arithmeticians, or calculators, and the algebraists. The algebraists devoted their attention to more abstract mathematics than the arithmeticians, or calculators, who emphasized the applied side of mathematics. By far, there was a lot more activity in applied mathematics than in abstract or pure mathematics.

That emphasis was largely due to the need of Hispanic society to maintain its imperial power in Europe and in the New World. The Spanish empire demanded an applied science and technological infrastructure that could enable the crown to generate enough economic and military resources to keep its dominance. Commerce, navigation, mining, and metallurgy, for example, were essential economic activities for Spain that demanded applied science and technology. The mathematical documents published in colonial Latin America introduced topics particularly adapted to this region.

Accordingly, the first mathematical works published in the New World in the sixteenth century addressed the applications of mathematics to commerce and mining. The earliest mathematics book in post-Columbian America was published in 1556 in México by Juan Diez Freyle. Its title was *Sumario compendioso de las quentas de plata y oro en los reynos del Perú son necesarias a los mercaderes: y todo genero de tratantes. Con algunas reglas tocantes al Arithmetica.* This work addressed the need for applied mathematics in commerce and mining and was used as a manual of practical arithmetic.

In 1597 Joan de Belveder published, in Lima, Perú, *Libro general de las reducciones de plata y oro*. Shortly thereafter, Francisco Garrequilla published a similar work in México. Interest in mathematics continued on through the seventeenth century in colonial Latin America.

The first treatise on arithmetic published in the New World is the *Arte para aprender todo el menor de la Arithmetica sin maestro* by Pedro Paz, which appeared in México City in 1623. Later on in that century, the Mexican-born Carlos de Siguenza y Gongora (1645-1700) distinguished himself as an archaeologist, astronomer, and mathematician. His investigations in those disciplines won him fame in México and Europe (Parkes 1966, p. 112; and Meyer and Sherman 1969, p. 226).

In the first half of the eighteenth century, Don Pedro de Peralta Barnuevo distinguished himself in mathematics in Perú. He held the chair of mathematics at the University of San Marcos in Lima. Throughout the colonial period, the Jesuits played an influential role in the intellectual life of Latin America. Their influence was felt in all academic disciplines including mathematics. But in 1767 the Spanish crown expelled the Jesuits from Spain and its colonies.

As a result, Latin America lost hundreds of scholars, teachers, and well-trained intellectuals. Unfortunately, the tradition of intellectual activity supported and fueled by the Jesuits was suddenly brought to a halt. It took more than 100 years for Latin America to recover from this blow due to the wars of independence and civil wars that raged in the region all through the nineteenth century.

Mathematical activity increased throughout Latin America in the twentieth century. For instance, Alberto P. Calderon, born in Argentina and still alive today, developed new theories and techniques in classical and functional analysis. Although he now lives in Argentina, Professor Calderon worked at the University of Chicago for many years. He was awarded the National Medal of Science in the United States and is a member of the prestigious Academy of Sciences of the United States, France, and Argentina. He is also a member of the Royal Academy of Sciences of Spain.

Currently, there is a lot of activity in applied and abstract mathematics in México, particularly at the National Autonomous University (UNAM) and the National Politechnic Institute (Poli), both in México City, and at the Politechnic Institute of Monterrey in northern México. Mathematics education has also experienced quite an impetus at these academic institutions and elsewhere in México. Mathematicians and mathematics educators there promote their professional development through their respective organizations, the Mexican Mathematical Society, the Mexican Mathematical Education Society, and the Mexican Council of Teachers of Mathematics.

Elsewhere in Latin America, there is a flurry of activity in both mathematics and mathematics education. Professionals in these fields come together in national, regional, and international conferences. For example, mathematics educators from México, Central America, and the Caribbean hold joint annual conferences. Every four years, mathematicians and mathematics educators from throughout Latin America, and the United States convene in the Inter-American Conference in Mathematics Education. This inter-American gathering is usually funded by the United Nations and meets in a different country every time it takes place.

LATINO MATHEMATICIANS IN THE UNITED STATES

Today there is a relatively large number of scientists, engineers, architects, and mathematicians in Latin American but there is a severe underrepresentation of Latinos in mathematics-based careers in the United States. While many of the professionals in Latin America have attained impressive accomplishments in their fields, we do not focus our attention on them in this essay because we do not have the information available to do so. Rather, we focus our attention on a few professionals in mathematics and mathematics education of Latino descent in the United States.

Statistical information about Latinos in mathematics-based careers in the United States prior to the 1970s is either non-existent or difficult to obtain. As governmental and educational agencies become more aware of the increasing percentage of Latinos in the U.S. population, the underrepresentation of this group in mathematics-based careers is becoming more widely known. The National Science Foundation (NSF) and other federal agencies have documented the phenomenon in many of their publications. Those interested in knowing more specific details about this issue may contact NSF in Washington, D.C.

Many Latinos in mathematics-based careers in the United States are active members in Latino and non-Latino professional organizations. The Society of Hispanic Professional Engineers (SHPE) and the Society for the Advancement of Chicanos and Native Americans in Science (SACNAS) are two professional organizations dedicated to promoting the participation and increasing the representation of Latinos in engineering, science, and mathematics-related fields. The following pages outline general biographical and professional data about some Latinos in mathematics and mathematics education who have provided their resumes to this author. As we shall see, several of these professionals are members of both SACNAS and non-Latino professional organizations.

The brief biographical sketches below show that Latinos have made contributions to modern abstract and applied mathematics. Specifically, they have contributed to the development of abstract and applied algebra, applied mathematics, complex analysis, computer applications, differential equations, functional analysis, mathematics education, mathematical modeling, numerical analysis, statistics, topology, and other branches of mathematics. Currently, there are some recent Ph.D. graduates and graduate students who are specializing in fractals, a very new branch of mathematics. Their contributions to this branch remain to be seen.

Manuel P. Berriozabal

Manuel P. Berriozabal was born in San Antonio, Texas and received his Ph.D. in mathematics from UCLA in 1961. His accomplishments are many; but here we mention only a few. He has taught mathematics courses at Loyola University in Los Angeles, at UCLA, Tulane University, the University of New Orleans, IBM in Austin, Texas, and at the University of Texas at San Antonio, where he is currently teaching. Dr. Berriozabal has authored many articles on topology and mathematics education. His contributions have been cited in about seventy professional publications. Professor Berriozabal is the director and founder of the San Antonio Pre-freshman Engineering Program, which has been very successful in helping pre-college students excel in mathematics. Many of the students who have gone through the Prefreshman Engineering Program have eventually gone on to major in a mathematics-based fields in college.

Dr. Berriozabal has also administered numerous educational projects and chaired many committees in Texas and Louisiana. He has been an invited speaker and panelist in many professional and community conferences. Professor Berriozabal has received numerous awards from national and local organizations in recognition of his contributions to the mathematics education of Latinos. He is a member of the American Mathematical Society (AMS), Mathematical Association of America (MAA), the National Council of Teachers of Mathematics (NCTM), SACNAS and other professional organizations.

Joaquin Basilio Diaz

The late Joaquin Basilio Diaz was born in Arecibo, Puerto Rico in 1920. He received the B.A. degree in mathematics from the University of Texas in 1920 and the Ph.D. degree in mathematics from Brown University in 1945.

He began his successful academic career as an assistant professor of mathematics at Carnegie Institute of Technology in 1946. In 1947 he moved to Brown University as an assistant professor of mathematics. In 1950 he became a research associate of the Institute for Fluid Dynamics and Applied Mathematics at the University of Maryland; he was promoted to associate research professor in 1951 and research professor in 1956. He spent the next year as a visiting professor of mathematics at Massachusetts Institute of Technology.

In 1966 he left Maryland to chair the Department of Mathematics at the University of California, Riverside. He left Riverside the next year and went to Rensselaer Polytechnic Institute as professor of mathematics and Albert Einstein Professor of Science. During his distinguished career he performed editorial responsibilities for numerous American Mathematics Society publications. He was also a member of AMS council.

Alfinio Flores

Dr. Alfinio Flores was born in México City and received his Ph.D. in mathematics education from Ohio State University in 1985. He is currently teaching mathematics education at Arizona State University. He has also taught math or mathematics education courses at San Diego State University in California, the National University of México, the University of Guanajuato in Guanajuato, México, and Ohio State University. He is the author of more than forty articles on mathematics education published in North American and Mexican professional journals and publications.

Professor Flores is a member of NCTM, MAA, AMS, School Science and Mathematics Association, Sociedad Matemática Mexicana, and other professional organizations. He has made presentations at numerous international, national, and local conferences on mathematics education.

Alba Gonzalez

Alba Gonzalez Thompson was born in Havana, Cuba and received her Ed.D. in mathematics education from the University of Georgia. She is currently teaching mathematics at San Diego State University and has taught at Illinois State University and in elementary and junior high schools in the United States and in the Dominican Republic. Professor Gonzalez Thompson is the author or co-author of over twenty articles and chapters in books. She has made numerous presentations in local, national, and international conferences on mathematics education.

Dr. Gonzalez Thompson is a member of NCTM, American Educational Research Association (AERA), International Group for the Psychology of Mathematics Education, and several other professional organizations. She has made valuable contributions to mathematics education as a researcher and as a practitioner.

Richard J. Griego

Richard J. Griego was born in Albuquerque, New México and received his Ph.D. in mathematics from the University of Illinois in 1965. He specializes in stochastic processes and has taught mathematics and statistics at the University of California, Riverside, Instituto Politecnico Nacional in México City, the University of California, Berkeley, the University of Utah, and the University of New México. He is currently the chairman of the Mathematics Department at Northern Arizona University, Flagstaff.

Dr. Griego has worked as a research scientists for projects on space medicine and has been a visiting scientist at the Los Alamos National Laboratory. Professor Griego has been the chairman of the Mathematics Department at the University of New México and has performed numerous other administrative duties in academic settings.

Professor Griego has made valuable contributions to theoretical and applied statistics. One of his contributions to applied statistics is cited in the *Encyclopedia Britannica*. Dr. Griego is very active in conferences and projects at the national and international levels. He has collaborated extensively with mathematicians in México on statistics. Professor Griego has received well over $5 million in grants for research and intervention programs. He has been the recipient of numerous research fellowships and honorific mentions in recognition of his academic accomplishments and his community service. Dr. Griego is a member of the American Mathematical Society, American Association for the Advancement of Science, Sociedad Matemática Mexicana, SACNAS, and other professional organizations.

Cleopatria Martinez

Cleopatria Martinez was born in Denver, Colorado and received her Ph.D. in mathematics education from the University of Colorado in 1985. Dr. Martinez is currently teaching mathematics at Scottsdale Community College in Scottsdale, Arizona. She has also taught mathematics at Metropolitan State College in Denver, at the Community College of Denver, and in the Denver Public Schools. Dr. Martinez is active in community and professional organizations. She is a member and past officer of SACNAS.

Daniel G. Martinez

Daniel G. Martinez was born in Colton, California and received his Ph.D. in mathematics from UCLA in 1971. He specializes in complex analysis and statistics. He is presently a mathematics professor at California State University, Long Beach, and he has served on several university committees. Dr. Martinez has also served as president of the Southern California Chapter of the American Statistical Association.

Roberto A. Mena

Roberto A. Mena was born in Yucatán, México and received his Ph.D. in mathematics from the University of Houston in 1973. Dr. Mena is currently a mathematics professor at California State University, Long Beach and has taught at the University of Wyoming, Ohio State University, Emory University, and the California Institute of Technology.

Professor Mena is the author of numerous articles on abstract and applied algebra. He has been the chairman of the Mathematics Department at California State University, Long Beach, and has made many presentations in professional conferences. Dr. Mena is a member of the Mathematical Association of America.

Luis Ortiz-Franco

Author Luis Ortiz-Franco was born in Jalisco, México and received his Ph.D. in mathematics education from Stanford University in 1977. He is currently a mathematics professor at Chapman University in Orange, California. He is the author of articles on mathematics education and is co-author of books and articles in the social sciences. Dr. Ortiz-Franco previously worked for the National Institute of Education (NIE) of the U.S. Department of Education, where he was an advocate for increasing funding to expand the the base of knowledge about the teaching and learning of mathematics among

Latinos. He has made numerous presentations in professional conferences and has organized several conferences on Latinos and mathematics.

Professor Ortiz-Franco is the third vice-president of the International Study Group on Ethnomathematics and frequently writes articles for the group's newsletter. He is a member of the Mathematical Association of America, the National Council of Teachers of Mathematics, SACNAS, and other professional organizations.

Tomas E. Salazar

Tomas E. Salazar was born in New México and received his Ph.D. in mathematics from the University of New México in 1976. He specializes in abstract algebra and has expertise in computer applications to educational and information processing environments. He also works on several areas of applied mathematics.

Dr. Salazar is currently the director of the Science Education Resource Center at the New México Highlands University. He has taught mathematics at the high school level and at the University of New México, at Fort Lewis College in Durango, Colorado, and at the Albuquerque Academy in Albuquerque, New México.

Professor Salazar was nominated for the outstanding teacher award at Fort Lewis College in 1977-78 and 1978-79. He has also received grants and fellowships from the National Science Foundation.

David A. Sánchez

David A. Sánchez was born in San Francisco, California and was vice president and provost of Lehigh University in Bethlehem, Pennsylvania. He received his Ph.D. in mathematics from the University of Michigan in 1967 and has taught mathematics at the University of Chicago, Manchester University in England, UCLA, Brown University, the University of Wisconsin, the University of New México, and the University of Wales. Dr. Sánchez has a distinguished career in teaching, research, and academic and public administration.

Dr. Sánchez was named assistant director of the National Science Foundation from 1990 to 1992 in recognition of his contributions to applied mathematics. Currently, he is deputy associate director for research and education at the Los Alamos National Laboratories. He has made valuable contributions in the areas of differential equations, functional analysis, numerical analysis, and mathematical modeling. He has published extensively in professional journals and has been the author or co-author of three books on differential equations.

Throughout his professional career, Dr. Sánchez has been active in efforts to increase the representation of Latinos in mathematics-based careers; he has written articles and made many presentations on this topic. He is a member of the American Mathematical Society, Mathematical Association of America, SACNAS, and other professional organizations. He has also collaborated extensively with mathematicians in Europe, México, and other parts of Latin America.

Richard A. Tapia

Richard A. Tapia was born in Santa Monica, California and received his Ph.D. in mathematics from UCLA in 1967. He is currently a professor of mathematics at Rice University in Houston, Texas and has taught mathematics at UCLA, the University of Wisconsin, Stanford University, and Baylor College of Medicine. He has a distinguished record in teaching, research, and community service.

In 1992 Professor Tapia was named a member of the prestigious National Academy of Engineering in recognition of his many valuable contributions to applied mathematics. He is co-author of two books and is the author or co-author of over sixty-three articles in professional journals on applied mathematics. In addition to his tireless efforts to increase the participation and representation of Latinos in mathematics-based careers here in the United States, Dr. Tapia has collaborated extensively with mathematicians in México.

Professor Tapia is a member of the American Mathematical Society, the Mathematical Association of America, the Society for Industrial and Applied Mathematics, SACNAS, and other professional organizations. He has made numerous presentations in conferences of these professional groups. Since 1972 Dr. Tapia has received support totalling millions of dollars for more than fifteen research projects. Some of his research has contributed to the development of modern space science.

William Y. Velez

William Y. Velez was born in Tucson, Arizona and received his Ph.D. in mathematics from the University of Arizona in Tucson. He specializes in abstract algebra and is an avid admirer of the mathematical accomplishments of pre-Columbian civilizations in México. He is currently a professor of mathematics at the University of Arizona and is at the National Science Foundation in Washington, D.C. Professor Velez also collaborates extensively with Mexican mathematicians on abstract mathematics.

Dr. Velez is the author of over thirty-five articles on abstract algebra published in both national and international journals. He has given numerous presentations in professional conferences and has presented many colloquia in universities in the United States, Germany, México, Canada, and the People's Republic of China. Professor Velez is also an untiring advocate for increasing the representation and participation of Latinos in mathematics-based careers; he has made many presentations and written articles on this topic.

Professor Velez is a member of the Board of Directors of SACNAS and is also a member of the Mathematical Association of America, the American Mathematical Society, the Sociedad Matemática Mexicana, and other professional organizations. Dr. Velez has three patents in signal processing, jointly with other scientists.

CONCLUSION

As this essay makes evident, Latinos have an illustrious history in mathematics. From the time of the Olmecs to today, their mathematical ideas range from creating unique number systems to making contributions to modern abstract algebra and topology. Their mathematics have built roads, cities, pyramids, and unique calendric systems, and have furthered modern space science. Thus, the mathematical power of Latinos has shaped the ground where they have walked and has explored the heavens. Their collective mathematical talents in applied and pure mathematics have fertilized mathematical thought for thirty-two centuries.

Now, official U.S. Census data indicate that the white population in the United States is becoming increasingly older while the non-white population, especially Latinos, is very young. These demographic trends have generated concern in public and private sectors about replacement of the cadres of scientists and engineers who will retire within the next twenty years. Government and industry can no longer rely solely on the white population to supply the scientists and technicians needed to keep the United States competitive in the international world of the twenty-first century. Minorities must be trained to accomplish this vital national objective.

The underrepresentation of Latinos in mathematics-based careers is a national problem. As described in this essay, Latinos have been performing sophisticated mathematics since 1200 B.C. We must all work together to overcome the obstacles that have prevented Latinos in the U.S. from realizing their potential in mathematics-based careers.

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