PPS
TI-83 Activities
Advanced Algebra 3-4

Teacher Notes

It is an expectation in Portland Public Schools that all teachers are using the TI-83 graphing calculator in meaningful ways in Advanced Algebra 3-4. This does not mean that students use a graphing calculator on every problem in the book, but all students deserve to have the opportunity to gain a deeper understanding of algebraic concepts which a graphing calculator can provide. The purpose of this set of activities is to provide teachers and students calculator keystrokes for specific problems in the McDougal Littell *Algebra 2 Explorations and Applications* textbook.

The activities are designed with the idea that teachers can use them for notes as they perform demonstrations/lectures for students, or students can use the sheets directly with a partner or in small groups as a classroom activity, or even as homework if students have access to a TI-83 calculator for homework. Some of the activities help students visualize concepts. Many of the activities model the use of multiple representations. Some of the activities model using the calculator to check predictions. Many screen dumps from the calculator are included because some people prefer visual cues to verbal ones.

For your convenience, a Table of Contents is provided. Information on the section, the actual problems, examples, or exploration activities being used, and a short description of how the calculator is being used is listed in the Table of Contents.

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4.5 Using Logarithms to Model Data

Example 1

Calculator: Logarithms for a list of values is calculated simultaneously. Linear regression for different combinations of lists is used.

7.1 Systems of Equations

Example 1

Calculator: Graphical solutions to systems of equations is found using the **intersect** feature. TABLE is used to determine an appropriate WINDOW for the graph.

7.3 Solving Linear Systems with Matrices

Example 1

Calculator: Matrix multiplication of an inverse matrix to solve a matrix equation.

10.3 Exploring Recursion

Example 3

Calculator: Using the recursive nature of the calculator on the calculation screen. Graphing in sequence mode to visualize sequence points (n, t_n).

15.1 Sine and Cosine on the Unit Circle

Example 1

Calculator: Graphical solutions finding intersections for simple trig functions.

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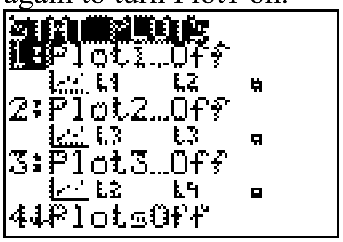
Lesson 1.2 Using Functions to Model Growth

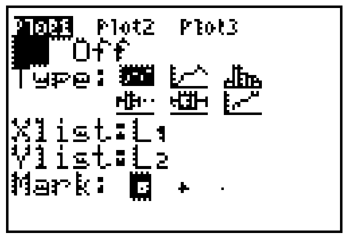
Example 2

p 11 Example 2:
Plot the data points for 1961, 1971, 1981, and 1991.

NOTE: The ordered pairs are (0,7), (1,21), (2,40), and (3,63). The x-coordinate represents the number of decades since 1961 and the y-coordinate represents plastics production in billions of pounds.

Step 1: Press **STAT PLOT** (2^{nd}, Y=). Press **ENTER** to access Plot 1 and press ENTER again to turn Plot1 on.



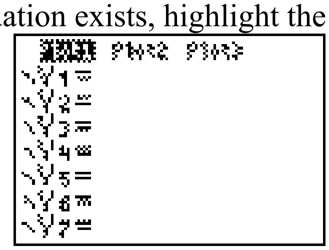


Be sure the icon for a scatter plot is highlighted under TYPE:, choose L_1 for Xlist and L_2 for Ylist. You may choose any mark by moving the blinking cursor onto the mark you choose and pressing ENTER.

Step 2: Be sure all other plots are off.

One way to do this is to press Y=. Any plots turned on are highlighted. If you need to turn off Plot2 or Plot 3, move to them with the directional arrows and press ENTER. By pressing ENTER, the Plot toggles off and on.

Step 3: Clear Y=. (If an equation exists, highlight the equation and press CLEAR.)

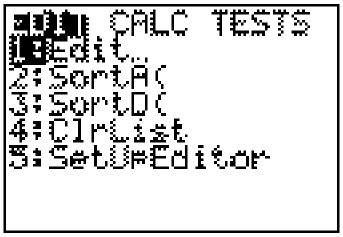


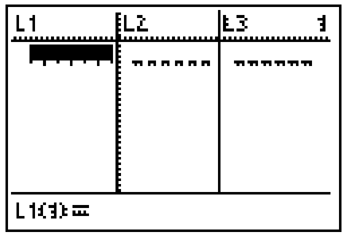
Step 4: Enter the data values.

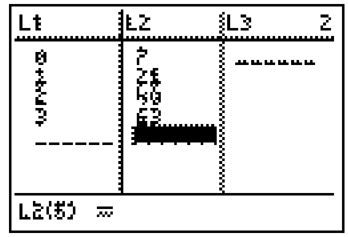
Press **STAT** and select **1:Edit…**

Clear the Lists by highlighting L_1 (or L_2) and pressing CLEAR followed by ENTER.

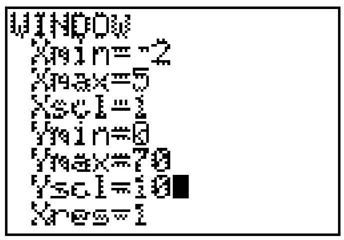
Enter the data values for x in L_1, press the right-arrow and enter the data values for y in L_2.



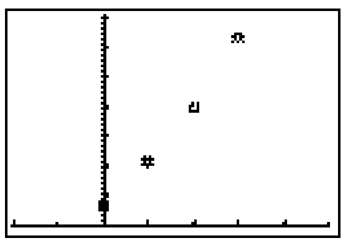




Step 5: Set the WINDOW. Since we know the data points, select window values that will show all of the points. Press **WINDOW** and enter the values below:

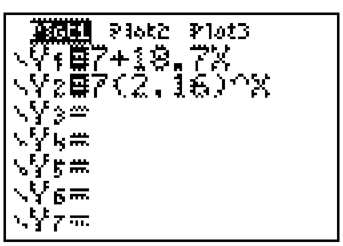


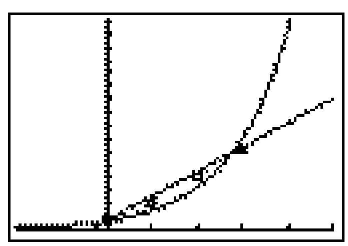
Step 6: Press **GRAPH** to see the scatter plot.



Step 7: Graph the equations y = 7 + 18.7x and y = $7(2.16)^x$, the linear and exponential models for this data.

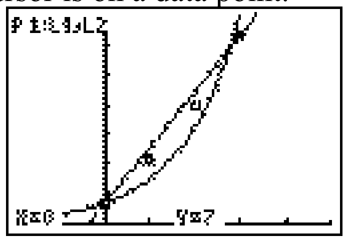
Press **Y=**. Enter 7 + 18.7X in Y_1 and $7(2.16)^x$ in Y_2. Press **GRAPH.**



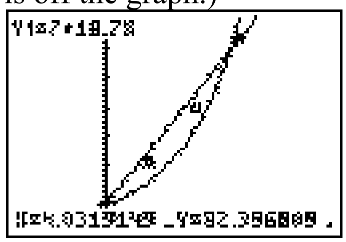


Step 8: Predict from each model what the plastics production will be in 2001 (when X = 4).

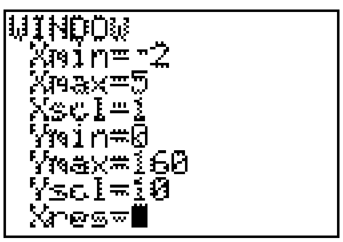
Press **TRACE**. Notice the cursor is on a data point.

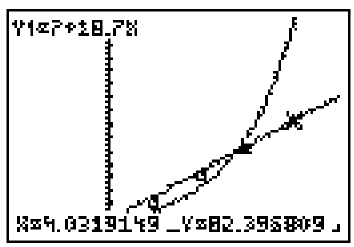


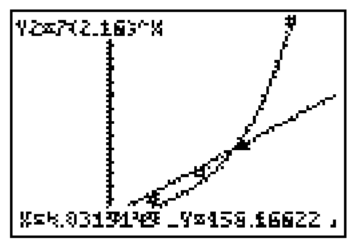
Press the down-arrow to move the trace to Y1=7+18.7X. Press the right-arrow until x = 4. (Notice the point is off the graph.)



Press **WINDOW**, change the window so that Ymax = 160, and press **GRAPH.** Repeat the trace and you see the point (4.03, 82.4). Press the down-arrow to see the value of Y2 = $7(2.16)^x$ when x = 4.03.







Does each equation reasonably model the data?

SUGGESTED PRACTICE:

p 13 E&A (9 – 12)
p 14 E&A (13, 14)
p 15 AYP (2)

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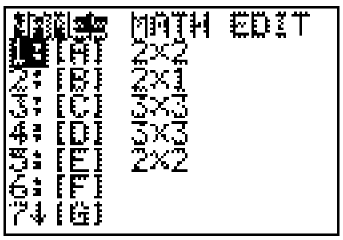
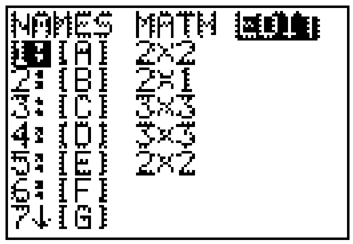
Lesson 1.4 Multiplying Matrices

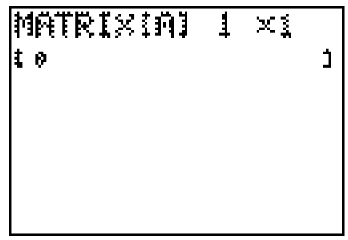
Example 2 – Method 2

p 24 Example 2 – Method 2:

Step 1: Enter the camping supplies order in matrix A.
Press **MATRX**. Press the right-arrow twice. With **EDIT** and **1:[A]** highlighted, press **ENTER.**

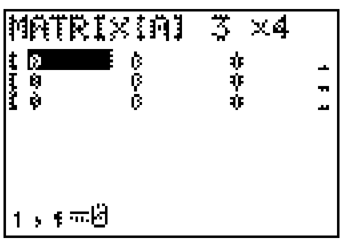


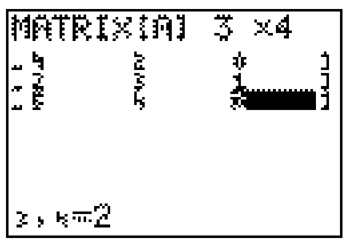




Your matrix[A] will look different if there is a matrix previously stored there.

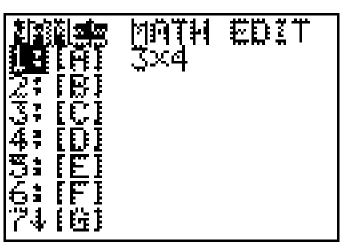
Enter the dimensions of matrix [A]. **3** (rows), **ENTER**, **4** (columns), **ENTER.**

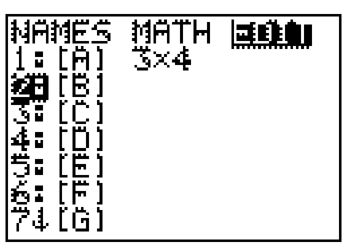


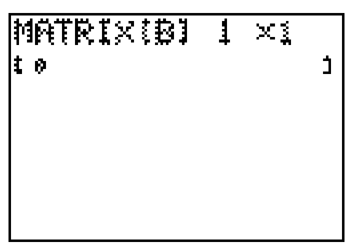


Enter the elements of [A]. Press **QUIT** (2^{nd}, MODE).

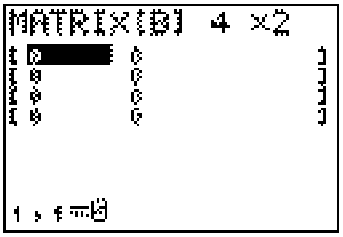
Step 2: Enter the shipping weight and cost of each item in matrix [B].
Press **MATRX**. Press the right-arrow twice and then the down-arrow once to highlight **EDIT** and **2:[B].** Press **ENTER.**

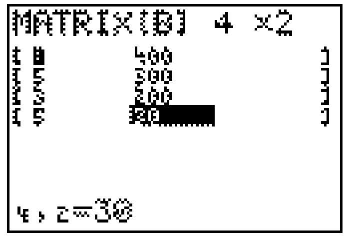






Enter the dimensions of matrix [B]. (4 x 2) and press **ENTER**. Enter the elements of matrix [B].

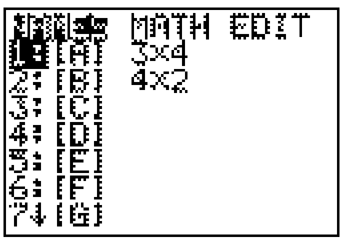


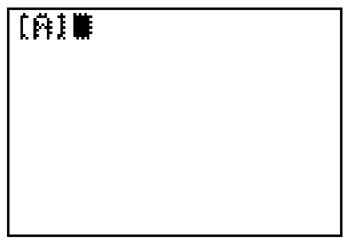


Press **QUIT** (2^{nd}, MODE).

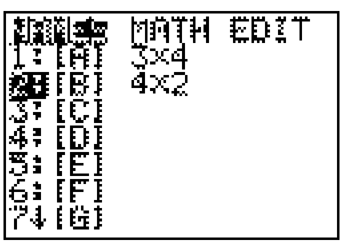
Step 3: Multiply [A][B].

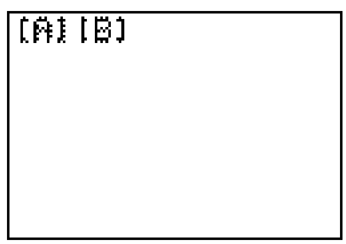
Press **MATRX**. With **NAMES** and **1:[A]** highlighted, press **ENTER**.



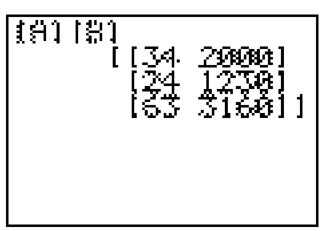


Press **MATRX**. Press the down-arrow once. With **NAMES** and **2:[B]** highlighted, press **ENTER**.





Press **ENTER** to multiply [A][B].



What do the numbers in the resulting matrix represent?

SUGGESTED PRACTICE:

p 25 – 28 E&A (6 – 17, 19, 23)

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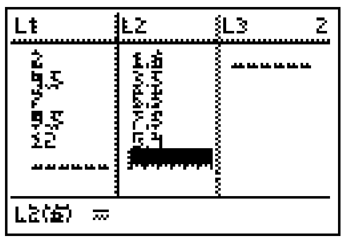
Lesson 2.1 Direct Variation

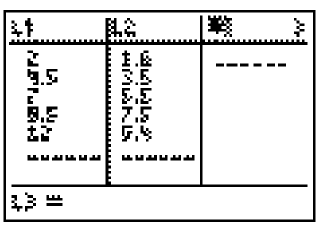
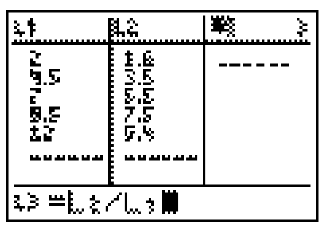
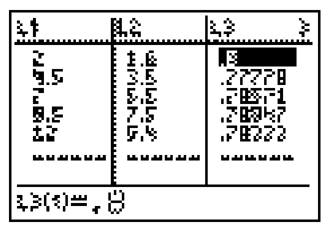
Example 1

p 45 Example 1:
When repeating steps to solve a problem (such as dividing speed by distance) you can use the TI-83 to perform the calculations in one step.

Step 1: Clear the statistics lists and enter distance in L1 and speed in L2.
Press **STAT**, **1** (to select 1:Edit…). If there are numbers in the lists use the arrow keys to highlight L1 (or any list to be cleared), press **CLEAR** and **ENTER**. Then enter the numbers in the two lists.



Step 2: Calculate the ratios of L2/L1. Use the arrows to highlight L3. Then type **L2,** ü**,** **L1,** and press **ENTER.**

Step 3: Check to see whether the data pairs have a constant ratio. The constant ratio in this case is approximately 0.79. Since $\frac{s}{d} = 0.79$, s = 0.79d. To find the speed, s, when the distance from the center of rotation, d, is 19.5, multiply 0.79(19.5) = 15.405.

SUGGESTED PRACTICE:
p 48 E&A (1 – 3)
p 51 E&A (28)

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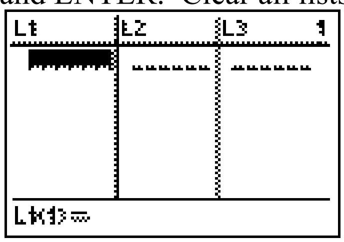
Lesson 2.4 Fitting Lines to Data

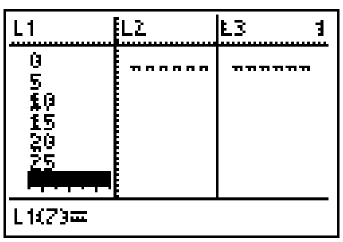
Example 2

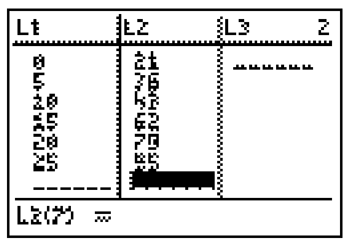
p 67 Example 2 – Using the bicycle data from the Example 1 on p 66:

Step 1: Clear the calculator's statistical memory. Enter the data as xy-pairs.
Press **STAT**, **1** (to select 1:Edit….). If the data lists L1, L2, or L3 contain numbers clear each on by using the arrow keys to highlight L1 (or L2 or L3), press CLEAR and ENTER. Clear all lists.

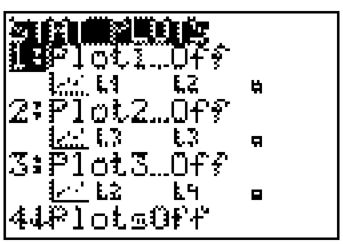


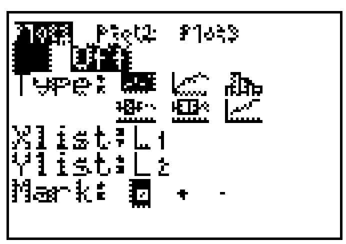
Enter the years since 1965 values in L1. Press the right-arrow and enter the number of bicycles (in millions) in L2.

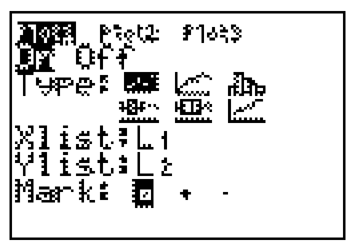




Step 2: Draw a scatter plot on an appropriate viewing window.
Press **STAT PLOT** (2^{nd}, Y=). Press **ENTER** to select Plot1 and press **ENTER** again to turn Plot1 on.

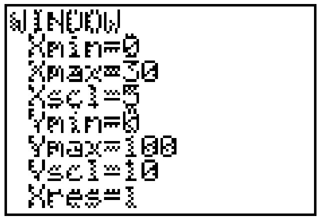


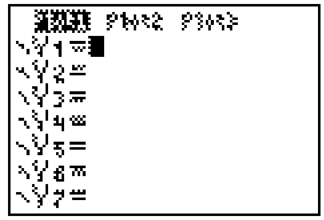




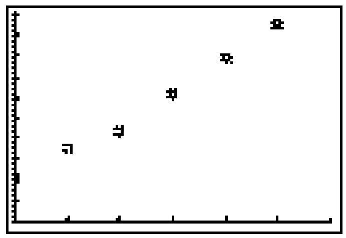
Make sure the same selections are chosen on your calculator. Set the window and clear the Y= so no graphs appear that you don't want to see.

Press **WINDOW** and enter the values below. Press **Y=** and CLEAR, ENTER for any Y with an equation.

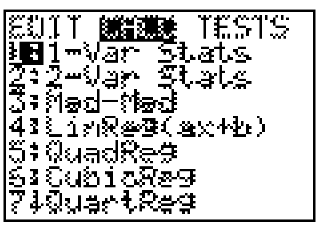


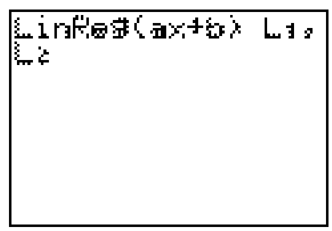


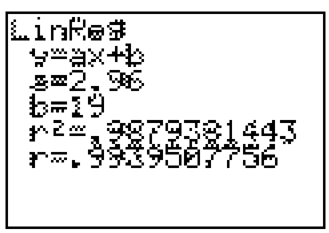
Press **GRAPH** to see the scatter plot.



Step 3: Have the calculator perform a linear regression (find the line of best fit.) Press **STAT** and use the right-arrow to highlight **CALC.** Press **4** (to select 4: LinReg(ax + b)). Then type **L1, L2** and press **ENTER.**



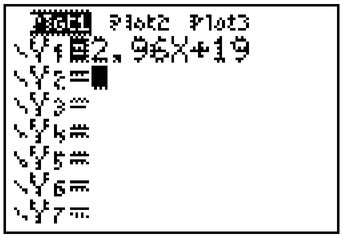


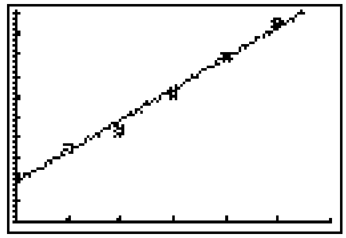


The calculator has determined that the equation of the line that best fits the data points is y = 2.96x + 19. NOTE: The number, r, is the correlation coefficient. The closer $|r|$ is to 1 the more linear the data.

Step 4: Graph the line of fit on the scatter plot from Step 2.

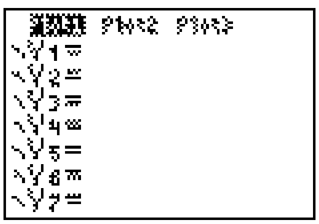
Option 1: Use the approximation:
Press **Y=** and enter 12.4x+31.6 into Y1. Then press **GRAPH**.

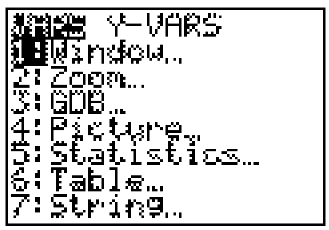


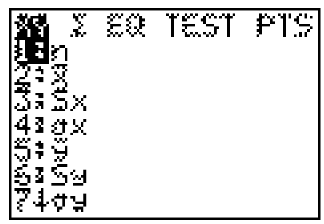


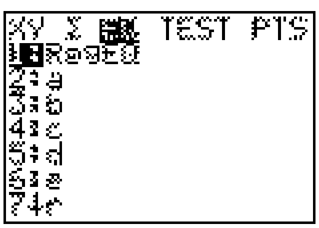
Option 2: Use the exact equation the calculator calculated. This equation is stored in a variable called RegEQ.

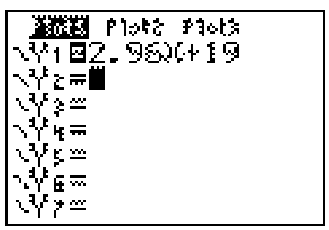
Press **Y=** and with Y1 cleared and highlighted, press **VARS, 5** (to select 5:Statistics). Press the right-arrow to highlight **EQ** and **1** (to select 1:RegEQ). The entire equation is stored in Y1. Press **GRAPH** to see the graph of the line and the scatter plot.

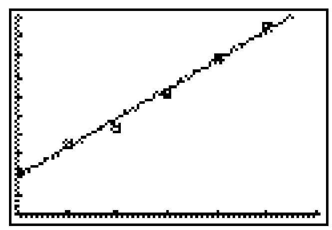




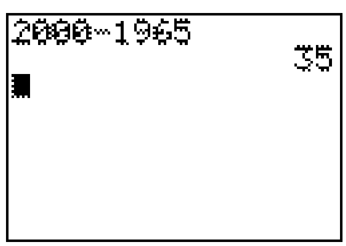




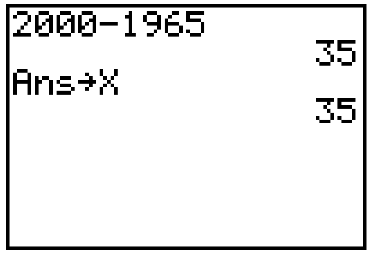




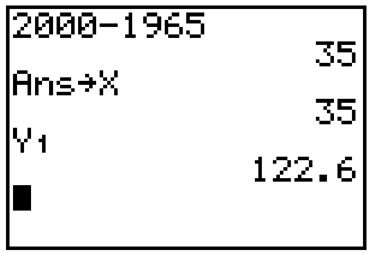
Step 5: Use the equation to predict the number of bicycles produced in the year 2000. Press **QUIT** (2^{nd}, MODE) and **CLEAR**. Type **2000 – 1965** and press **ENTER.**



We are looking for the value of y1 = 2.96x + 19 when x = 35.



Press **STO->**, **X, ENTER** to store 35 in x. To find the value of Y1 when x = 35, press **VARS** and the right-arrow to highlight **Y-VARS.** Select **1:Function** followed by **1:Y1.** Press **ENTER.**



SUGGESTED PRACTICE:
p 68 CKC (1, 2)
p 68 – 71 E&A (1, 2, 3, 5, 9, 10)

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Lesson 3.3 Graphs of Exponential Functions

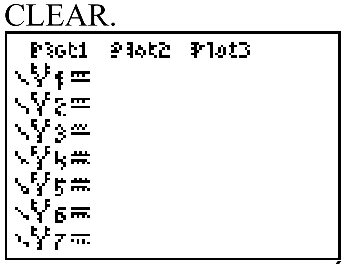
Exploration

P 107 EXPLORATION:

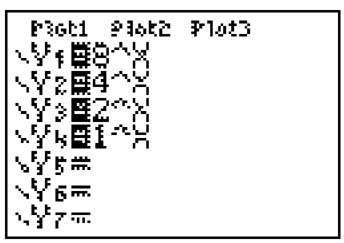
One of the most powerful uses of a graphing calculator is to determine how changing the values of certain numbers in an equation (called parameters) affect the graph of the equation. If you forget how a particular number affects a graph it is easy to test it out with a few examples on the calculator at any time.

Step 1: Graph $y = 8^x$, $y = 4^x$, $y = 2^x$, and $y = 1^x$ on the same window.

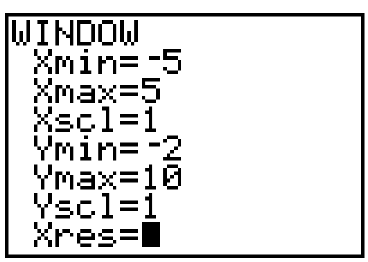
Press **Y=**. if any of the Plots at the top of the screen are highlighted, use the directional arrow keys to move the cursor to the Plot that is highlighted and press ENTER. This will turn the STAT PLOT off. Clear the Y= by highlighting each equation and pressing CLEAR.



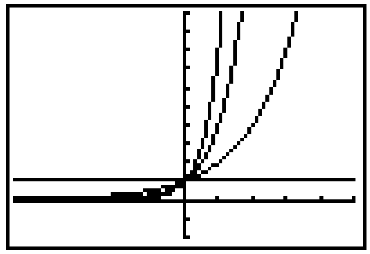
With the cursor in **Y1=** type **8, ^, X.** (Use the (X,T,θ,n) key for X.) Press **ENTER.** Similarly enter 4^X in Y2, 2^X in Y3, and 1^X in Y4.



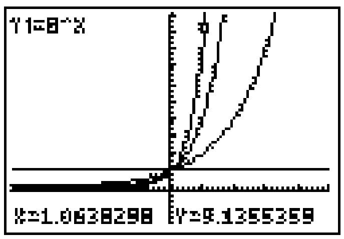
Set the viewing window. Press **WINDOW** and enter the following values:

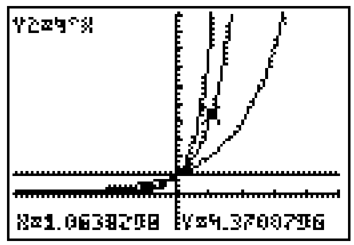


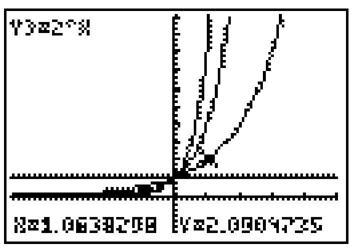
Press **GRAPH** to watch the equations graph.

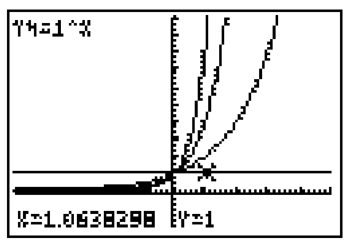


Press **TRACE** and then press the right-arrow until x = 1. (The cursor is tracing Y1 = 8^X.) Press the down-arrow to see the value of Y2 = 4^X when x = 1.06. Press the down-arrow again to see the value of Y3 = 2^X when x = 1.06. Press the down-arrow one more time to see the value of Y4 = 1^X when x = 1.06.





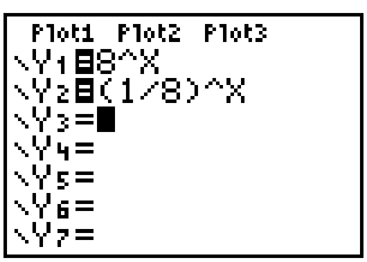


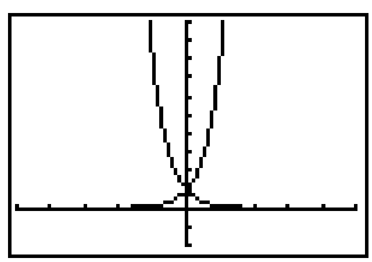


QUESTION 1: What do you notice about the y-intercepts of the graphs of these four equations?

Step 2: a) Graph $y = 8^x$ and $y = \left(\frac{1}{8}\right)^{x}$ on the same window.

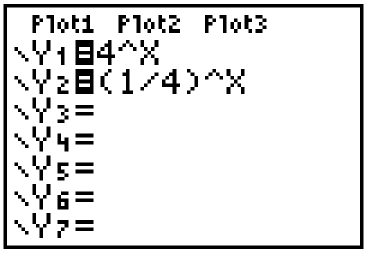
Press **Y=** and clear Y2, Y3, and Y4. Press the up-arrow until the cursor is on Y2. Type (1/8)^X. Press **GRAPH.**

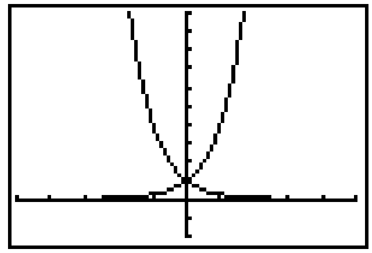




b) Graph $y = 4^x$ and $y = \left(\frac{1}{4}\right)^x$ on the same window.

Press **Y=**. Enter **4^X** in Y1 and **(1/4)^X** in Y2. Press **GRAPH.**

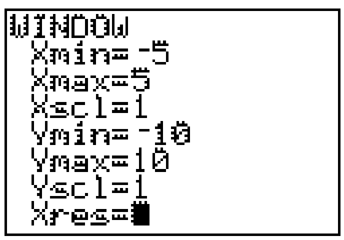




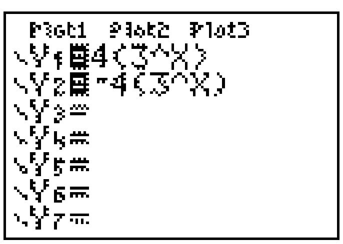
c) Graph $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$ on the same window.

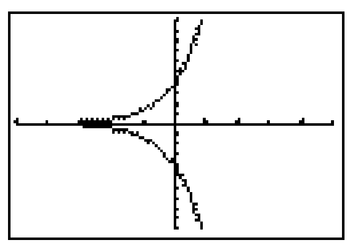
QUESTION 2: What do you notice about these pairs of graphs?

Step 3: a) Compare the graphs of y = 4(3^X) and y = -4(3^X).
Press **WINDOW** and enter the following values:



Press **Y=** and enter 4(3^X) in Y1 and –4(3^X) in Y2. Press **GRAPH.**





b) Compare the graphs of y = 5(3^X) and y = -5(3^X).

QUESTION 3: What is the effect on the graph of $y = ab^x$ when a is replaced by –a?

SUGGESTED PRACTICE:
p 110 – 113 E&A(1 – 9, 31, 32)

ANSWERS:
Question 1: The y-intercepts are all (0,1).
Question 2: They are reflections across the y-axis.
Question 3: The graph is reflected across the x-axis.

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TI-83 Activities
Advanced Algebra 3-4

Lesson 3.3 Graphs of Exponential Functions

Example 1

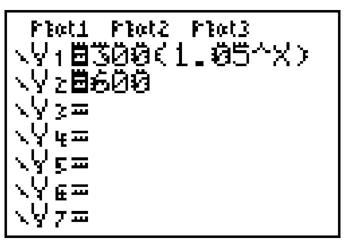
This technique for solving an exponential equation is important for students to understand. First, this technique can be used to solve any equation that can be entered on a graphing calculator. Second, it is important to understand what it means to solve an exponential equation and the usefulness of this solution before learning a symbolic way to solve exponential equations (using logarithms) in the next chapter.

p 108 Example 1:

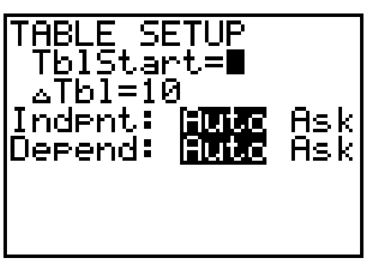
The amount in your account after x years will be $y = 300(1.05)^x$. The question is "How many years will it take to double your money?" Since y is the amount in your account after x years, the question becomes, "What value of x will make Y = 2(300) or 600?" In other words, we need to solve the equation $600 = 300(1.05)^x$ for x.

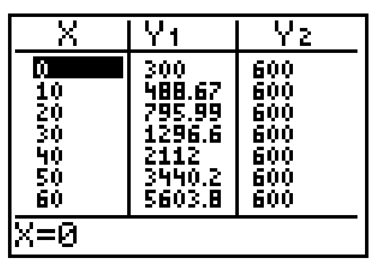
To do this graphically on the TI-83, we need to find the point of intersection of Y1 = 300(1.05^X) and Y2 = 600.

Step 1: Enter the two equations in **Y=.** (Be sure all Plots are off.)



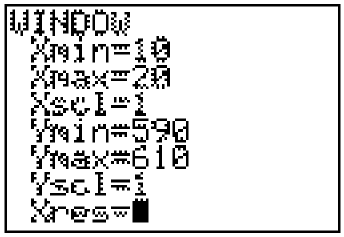
Step 2: Set the window so you will see the point of intersection. One way to determine an appropriate window is to use the TABLE. Press **TBLSET** (2nd, WINDOW) and enter the following values. Then press **TABLE** (2nd, GRAPH).



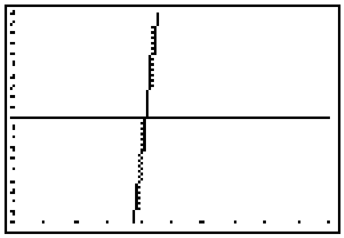


QUESTION 1: Between what two x values will Y1 = 600?

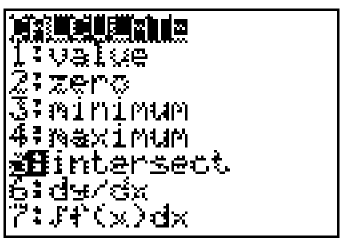
Press **WINDOW** and enter the values below:

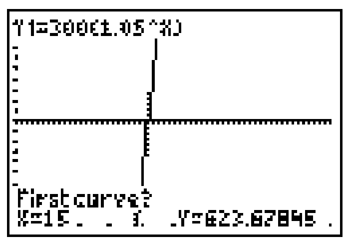


Step 3: Press **GRAPH**.

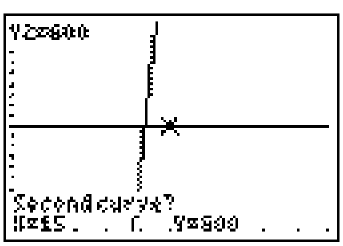


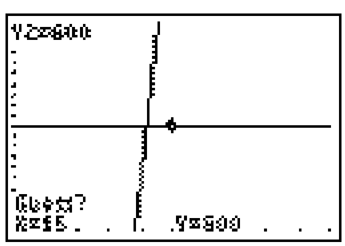
Step 4: Find the point of intersection.
Press **CALC** (2nd, TRACE) and select **5:intersect.**

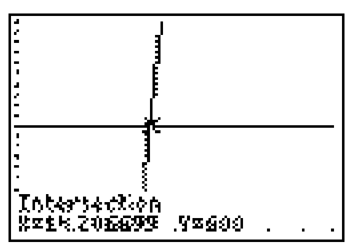




The TI-83 is asking you to select the first curve you want to find the intersection with. (There could be several curves graphed at once. Pressing the down- or up-arrows will scroll you through the list in Y=.) Notice Y1=300(1.05^X) is at the top of the screen. Press **ENTER** to select Y1 as the first curve.







Press **ENTER** to select to select Y2 = 600 as the second curve. Now the TI-83 is asking you to provide a guess for the point. The calculator needs a point to start its calculation. Press **ENTER** and the calculator will use the point where the cursor is as the Guess.

QUESTION 2: What is the meaning of this point of intersection?

At 5% interest compounded annually, the amount doubles every 14.2 years/

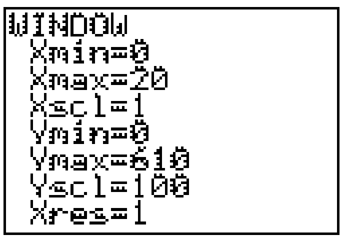
QUESTION 3: How much money will be in the account after 28.4 years?

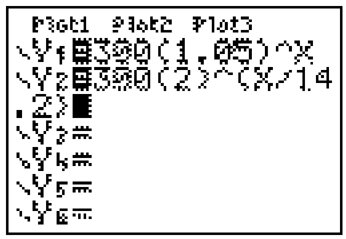
QUESTION 4: How much money will be in the account after 42.6 years?

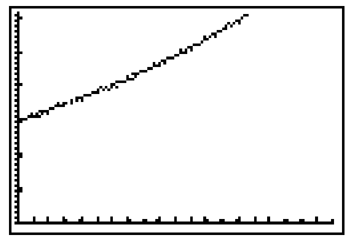
QUESTION 5: How much money will be in the account after X years?

The equation $y = 300(1.05)^x$ could be rewritten as $y = 300(2)^{\frac{x}{14.2}}$ since it doubles every 14.2 years.

To show that these equations are equivalent (disregarding round-off error), graph the two equations Y1 = 300(1.05)^X and Y2 = 300(2)^(X/14.2).

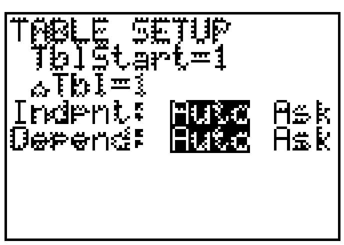


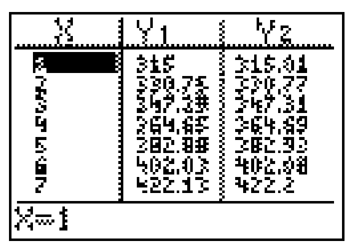




QUESTION 6: How do you know by looking at the graph that these two equations are equivalent?

Another way to check that the equations are equivalent is to look at some numerical values. Since the equations are already in Y=, press **TBLSET** (2nd, WINDOW) and enter the following values: Then press **TABLE** (2nd, GRAPH).





QUESTION 7: What do you notice about the values in the table?

SUGGESTED PRACTICE:

p 109 Example 2

p 110 CKC(10 – 12)

p 112 E&A(16 – 21, 24c)

ANSWERS:
Question 1: 10 and 20
Question 2: It will take about 14.2 years to double the $300.
Question 3: $1200
Question 4: $2400
Question 5: $300(2)^{\frac{x}{14.2}}$
Question 6: They are the same graph.
Question 7: Values in Y1 and Y2 are almost alike.

PPS
TI-83 Activities
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Lesson 3.4 The Number e

Replacement for EXPLORATION

p 115 Replacement for EXPLORATION:

When interest is compounded n times per year for t years at an interest rate r (expressed as a decimal), a principal (beginning amount) P dollars grows to the amount A given by this formula: $A = P\left(1+\frac{1}{n}\right)^{nt}$.

To investigate a very important number in mathematics, let's determine the amount of money in an account after 1 year (t = 1) if we invest $1.00 at 100% interest (r = 1).

$$A = 1\left(1+\frac{1}{n}\right)^{n(1)}$$

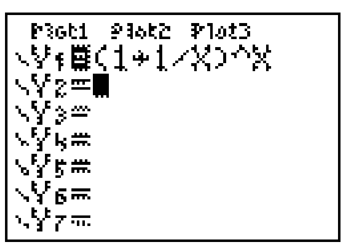
or

$$A = \left(1+\frac{1}{n}\right)^{n}.$$

If the interest is compounded annually (once each year), then n = 1.
If the interest is compounded quarterly (4 times per year), then n = 4.
If the interest is compounded monthly, then n = 12.
If the interest is compounded daily, then n = 365.

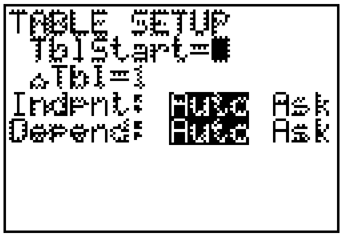
Since we are interested in investigating an equation for specific values of n that do not start at one value and go up by equal amounts, we will use the ASK feature of the TABLE.

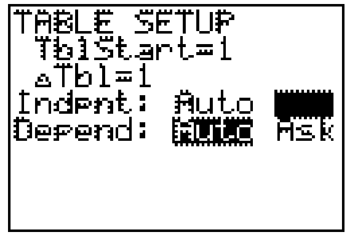
Step 1: Enter (1 + 1/X)^X in Y1.
Press **Y=.** Clear all plots. Type (1 + 1/X)^X in Y1, and press **ENTER.**



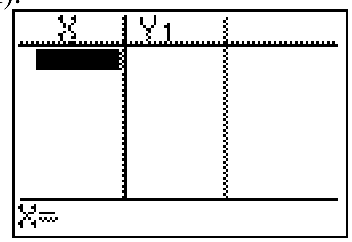
Step 2: Set up the TABLE.

Press **TBLSET** (2^{nd}, WINDOW). Use the directional arrows to highlight **ASK** under **Indpnt:**, and press **ENTER.** (Since Y1 depends on X, X is the independent variable.)

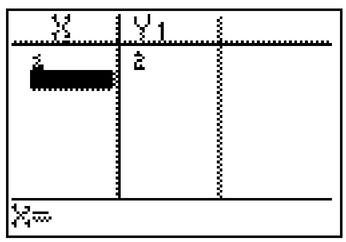




Step 2: Press **TABLE** (2^{nd}, GRAPH).

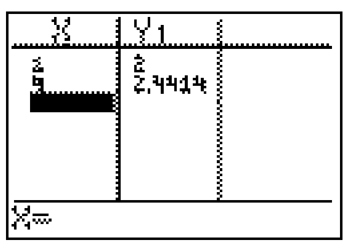


Notice the table is empty and the cursor is in the X column. Type **1** and press **ENTER.**

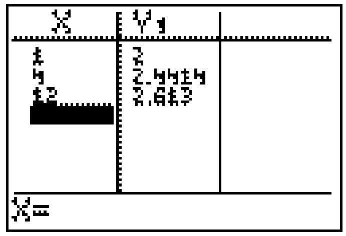


When x = 1, y = $\left(1+\frac{1}{1}\right)^{1} = 2$.

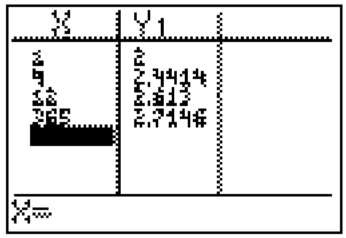
Type **4** and press **ENTER** to determine the amount after 1 year if the interest is compounded quarterly.



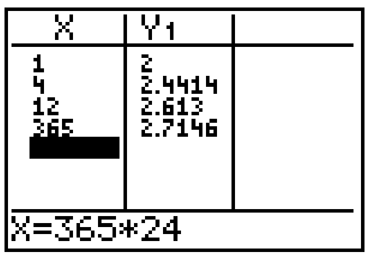
Type **12** and press **ENTER** to determine the amount after 1 year if the interest is compounded monthly.

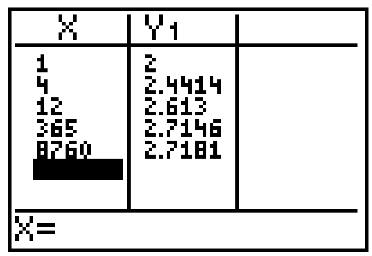


For interest compounded daily, type **365** and press **ENTER.**



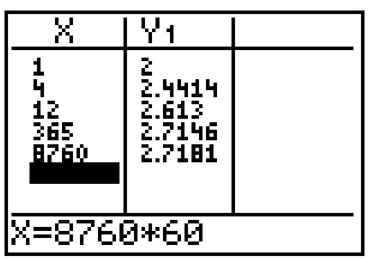
For interest compounded hourly, type **365*24** and press **ENTER.**

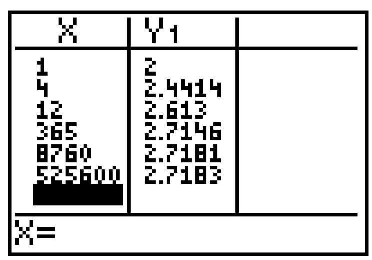




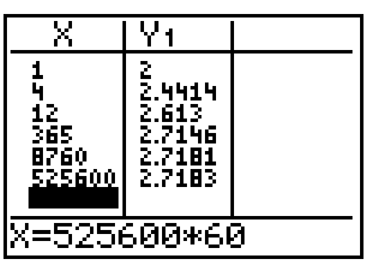
Notice that the TI-83 calculated 365 times 24 and entered 8760 in the X column.

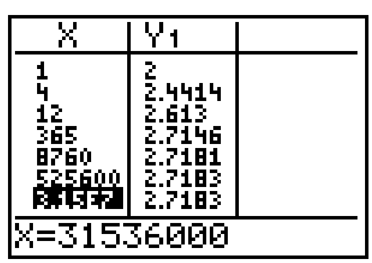
For interest compounded every minute, type **8760*60** and press **ENTER.**





For interest compounded every second, type **525600*60** and press **ENTER.**



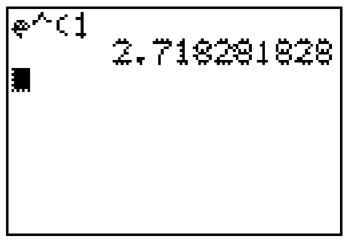


QUESTION: What seems to be happening to the value of Y1 = (1 + 1/X)^X as X gets larger?

As the frequency of compounding increases (as x gets larger and larger), the amount in the account approaches $2.718…

Because the number 2.718… is special in mathematics it was given the name **e**. (Leonard Euler first used e to represent 2.718… in 1727.)

e is often used as a base in exponential functions. Press **QUIT** (2^{nd}, MODE) to leave the table. Press **CLEAR** to clear the screen. Press e^x (2^{nd}, LN), **1, ENTER.**



e is an irrational number like π . The TI-83 is giving a decimal approximation for e rounded to 9 decimal places.

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TI-83 Activities
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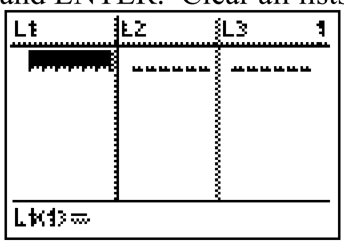
Lesson 3.5 Fitting Exponential Functions to Data

Example 3

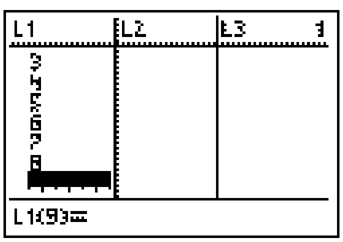
p 125 Example 3:

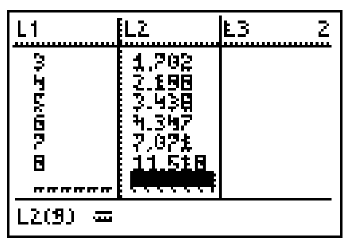
We have previously fit linear functions to data. Using the age and weight data of Atlantic Cod in Example 3, we can similarly fit an exponential function to data.

Step 1: Clear the calculator's statistical memory. Enter the data as xy-pairs.
Press **STAT**, **1** (to select 1:Edit….). If the data lists L1, L2, or L3 contain numbers clear each on by using the arrow keys to highlight L1 (or L2 or L3), press CLEAR and ENTER. Clear all lists.

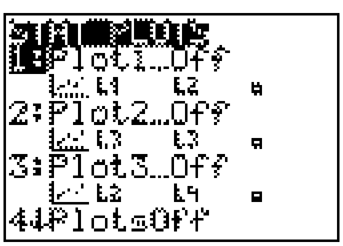


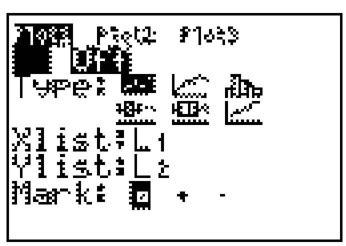
Enter the age in years in L1. Press the right-arrow and enter the weight in kg in L2.

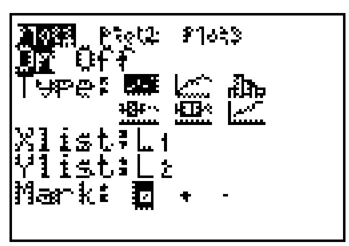




Step 2: Draw a scatter plot on an appropriate viewing window.
Press **STAT PLOT** (2^{nd}, Y=). Press **ENTER** to select Plot1 and press **ENTER** again to turn Plot1 on.

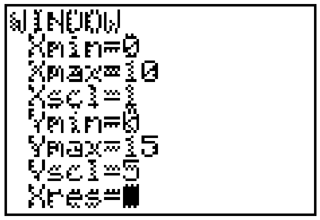


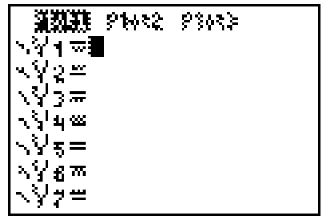




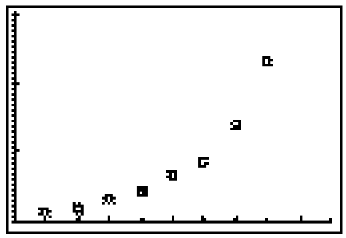
Make sure the same selections are chosen on your calculator. Set the window and clear the Y= so no graphs appear that you don't want to see.

Press **WINDOW** and enter the values below. Press **Y=** and CLEAR, ENTER for any Y with an equation.



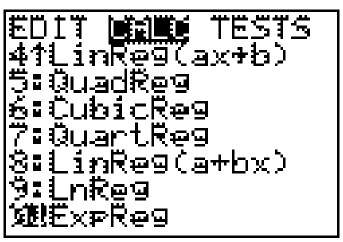


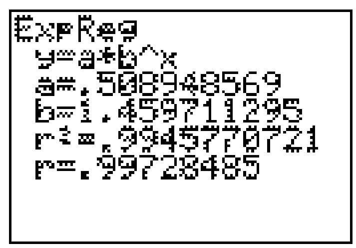
Press **GRAPH** to see the scatter plot.



Step 3: Use the exponential regression feature of the TI-83.

Press **STAT** and use the right-arrow to highlight **CALC.** Press the down-arrow until 0 Ø ExpReg is highlighted and press **ENTER.** Then type **L1, L2** and press **ENTER.**





The calculator has determined that the exponential function that models the Atlantic Cod weight data is

$$y = 0.59(1.460)^x$$

where x is age in years and y is weight in kg.

SUGGESTED PRACTICE:

p 126 E&A (11, 14)
p 129 AYP (5)

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Lesson 4.5 Using Logarithms to Model Data

Example 1

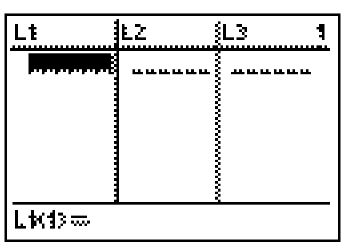
In this lesson, linear regression and what we know about logarithms is used to find exponential and power functions to model data. While there are more expedient ways to find these functions, the real value in this lesson is to discover the relationship between the linear function of log y as a function of x and the exponential function of y as a function of x. The calculator is used to find the logarithm of each data value in a list and for linear regression. After the calculator does its work, there is still plenty of algebra to do by hand.

p 169 Example 1:

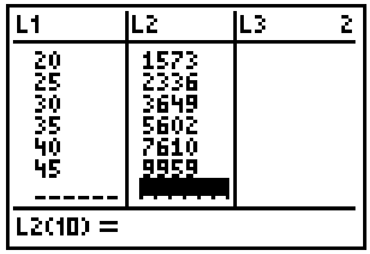
After World War II there were fewer than 300 cranes in the wetlands near the ocean in Izumi, Japan. The table shows how the number of cranes increased from 1945 to 1990.

t = years since 1945	P = crane population
5	293
10	299
15	438
20	1573
25	2336
30	3649
35	5602
40	7610
45	9959

Step 1: Clear the calculator's statistical memory. Enter the data as xy-pairs.
Press **STAT**, **1** (to select 1:Edit….). If the data lists L1, L2, or L3 contain numbers clear each one by using the arrow keys to highlight L1 (or L2 or L3), press CLEAR and ENTER. Clear all lists.



Enter the years since 1945 values in L1. Press the right-arrow and enter the crane population in L2.

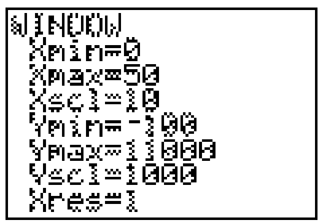


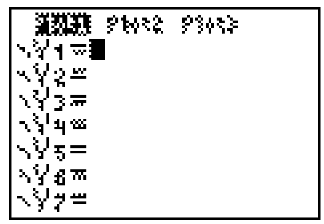
Step 2: Draw a scatter plot on an appropriate viewing window.

Press **STAT PLOT** (2^{nd}, Y=). Press **ENTER** to select Plot1 and press **ENTER** again to turn Plot1 on.

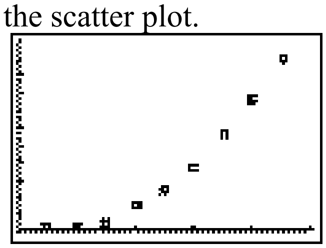
Set the window and clear the Y= so no graphs appear that you don't want to see.

Press **WINDOW** and enter the values below. Press **Y=** and CLEAR, ENTER for any Y with an equation.





Press **GRAPH** to see the scatter plot.

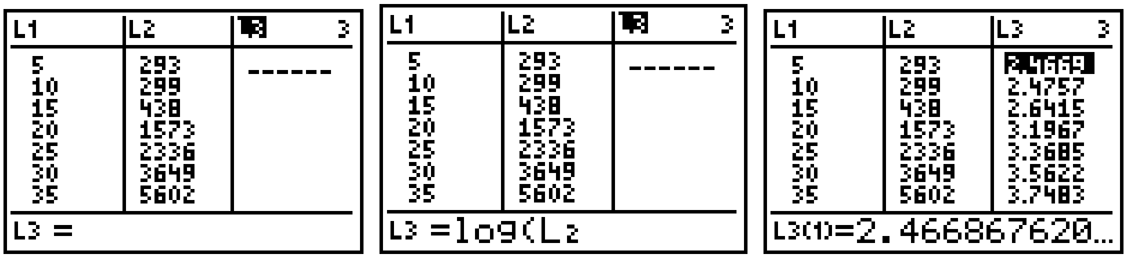


QUESTION 1: Do you think a linear function is a good model for the crane data? Why or why not?

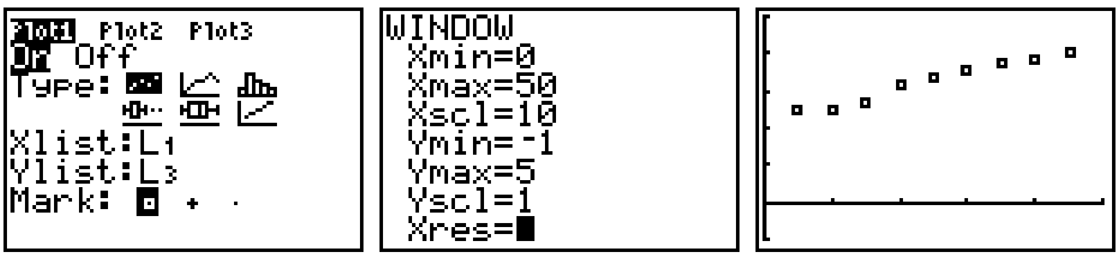
QUESTION 2: What type of function do you think best models the crane data? Explain.

Sometimes by taking the logarithm of the dependent variable or by taking the logs of both the independent and dependent variables, a linear equation is appropriate to model the resulting data. The TI-83 can find the log of each population value for our data currently in L_2 and store it in L_3.

Press **STAT** and select **1:Edit…** under **EDIT.** Use the directional arrows to highlight L_3 and press **LOG,** L_2 (2^{nd}, 2), **ENTER.**

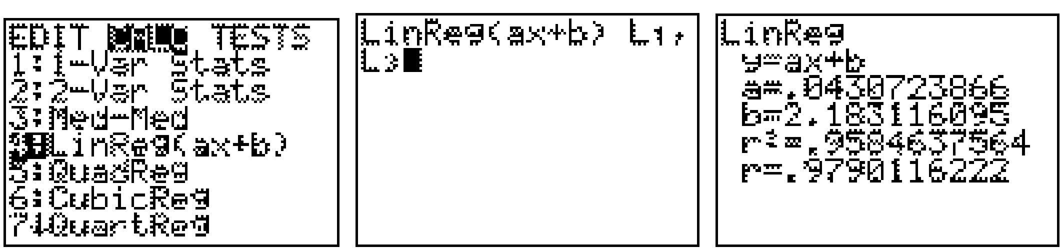


L_3 now contains the log values of the numbers in L_2. To see a scatter plot of (t, log P) or (L_1, L_3) press **STAT PLOT** (2^{nd}, Y=), **ENTER.** Use the down-arrow until **Ylist:** is highlighted and press L_3 (2^{nd}, 3). Change the WINDOW to see these data points, and press **GRAPH** to see the scatter plot.



QUESTION 3: Does there appear to be a linear relationship between t and log P?

To have the TI-83 calculator determine the line of best fit for the (t, log P) data points, press **STAT** and right-arrow to highlight **CALC.** Select **4:inReg(ax + b).** Type L_1, L_3 and press **ENTER.**



Therefore, the linear equation that models (t, log P) is

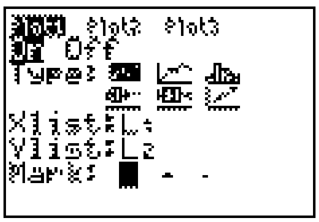
$$\text{LogP} = .0431t + 2.1831$$

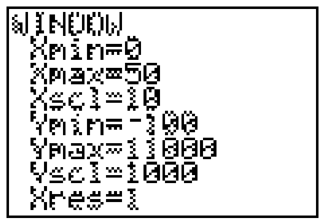
Solving this equation for P will give P as a function of t.

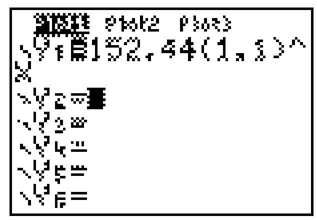
$$\text{Log}P = .0431t + 2.1831$$
$$10^{\log P} = 10^{.0431t+2.1831}$$
$$P = \left(10^{.o431t}\right)\left(10^{2.1831}\right)$$
$$P = \left(10^{2.1831}\right)\left(10^{.0431}\right)$$
$$P = 152.44\left(1.10\right)^{t}$$

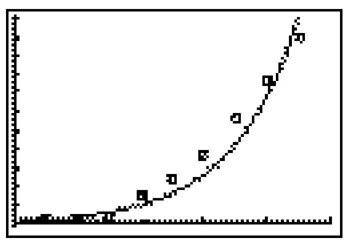
This is an exponential function. Let's go back and plot the original data points (t, P) with the graph of this exponential function to see if it is a good model for the crane data.

Press **STAT PLOT**, **ENTER**, and change **Ylist:** back to L_2. Change the WINDOW back. Enter Y1 = 152.44(1.1)^X uncer **Y=**. Press **GRAPH.**









QUESTION 4: Do you think the given function is a good model for the crane data? Explain.

QUESTION 5: Predict the number of cranes in Izumi in 1998.

QUESTION 6: Based on the exponential model you found, by about what percent did the Izumi crane population increase each year from 1945 to 1990?

SUGGESTED PRACTICE:

p 170 Example 2 (Be sure you understand the algebraic steps in this example.)
p 171 CKC (1 – 3)
p 172 – 3 E&A (8, 16, 17)
p 175 AYP (6)

ANSWERS:

Question 1: No, the data points follow a curving pattern.

Question 2: Exponential. The points are on a curve that goes up.

Question 3: Yes

Question 4: Yes, the points are fairly close to the curve with some above and some below.

Question 5: 23818

Question 6: 10%

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Lesson 5.2 Translating Parabolas

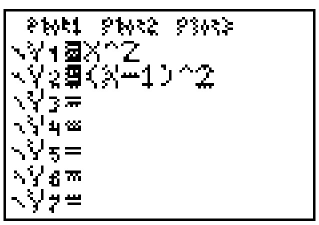
EXPLORATION

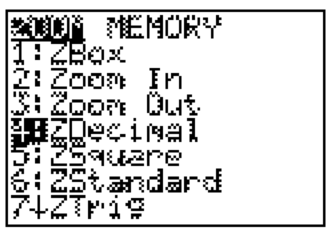
It is often helpful to use the graphing feature of a calculator to determine how changing the equation in a particular way changes the graph of the equation. You should also, however, be able to sketch graphs by hand. A useful technique for sketching simple parabolas is illustrated in Lesson 5.1 Example 2 (page 186). Combining this technique with what you discover in Lesson 5.2 about Moving Parabolas Around should allow you to sketch parabolas in the form $y = a(x-h)^2 + k$ quickly **by hand.**

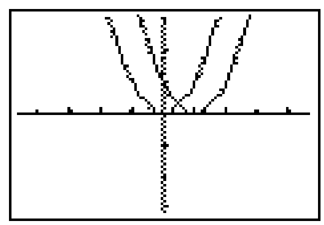
p 192 EXPLORATION:

Step 1: Compare the graphs of $y1 = x^2$ and $y2 = (x-1)^2$.

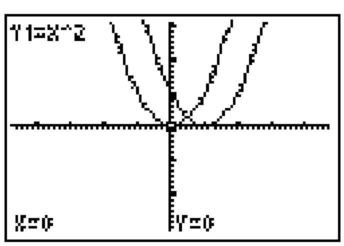
Press **Y=** and make sure all Plots are off. Enter **X^2** in **Y1** and **(X – 1)^2** in **Y2.** Press **ZOOM** and select **6:ZStandard** to view these graphs on the standard window. Then press **ZOOM**, and select **4:ZDecimal** to select a window that will trace in increments of 0.1.

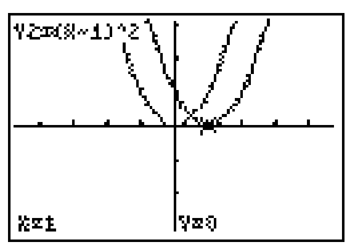






Press **TRACE** to trace Y1 = X^2. Notice that the trace is on the vertex in this case. Press the down-arrow t move to Y2 = (X – 1)^2, and press the right-arrow until the cursor is on the vertex of Y2 = (X – 1)^2.





QUESTION: How is the graph of $y1 = x^2$ related to the graph of $y2 = (x-1)^2$?

Refer back to the EXPLORATION on page 192 and repeat this process to compare the other pairs of equations.

NOTICE: In the Questions in the EXPLORATION you are asked to sketch a graph of each equation by hand, and then use the graphing calculator to check your sketch.

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Lesson 7.1 Systems of Equations

Example 1

p 291 Example 1:

The amount, A, owed on each loan after p monthly payments is given by:

The first loan option: $A = 109{,}688 - 97{,}688(1.00242)^{p}$

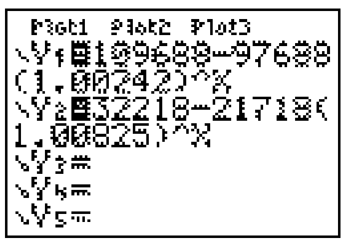
The second loan option: $A = 32218 - 21718(1.00825)^{p}$

One way to find the number of months when the amounts needed to pay off the two loans are equal, graph the two equations and find the point of intersection.

Step 1: Enter the equations to be graphed in Y=.

Press **Y=** and make sure no Plots are highlighted. (If a Plot is highlighted, use the arrow keys to move over it and press ENTER.)

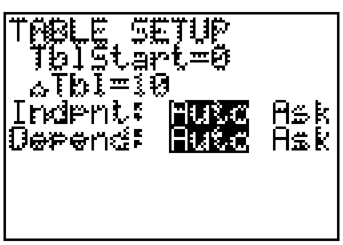
Enter 109,688 – 97,688(1.00242)^X in Y1 and 32218 – 21718(1.00825)^X in Y2.

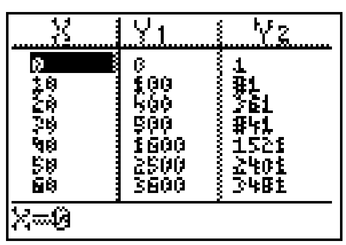


Step 2: Set an appropriate window.

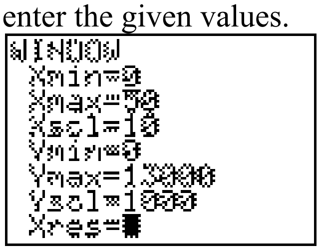
The TABLE can be used to determine appropriate WINDOW values.

Press **TBLSET** (2^{nd}, WINDOW) and enter the values given. Then press **TABLE** (2^{nd}, GRAPH).

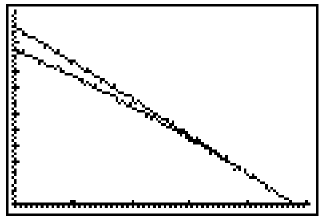




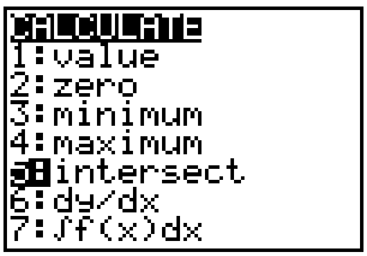
Press **WINDOW** and enter the given values.

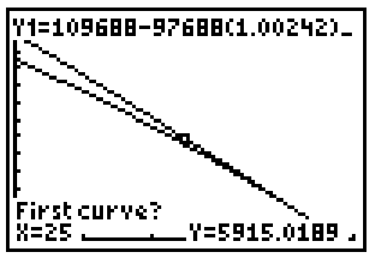


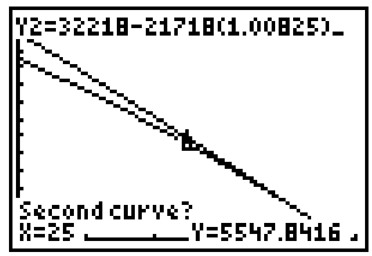
Step 3: Press **GRAPH** to see the graph.

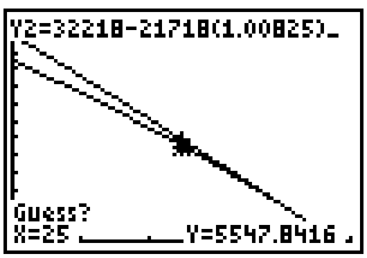


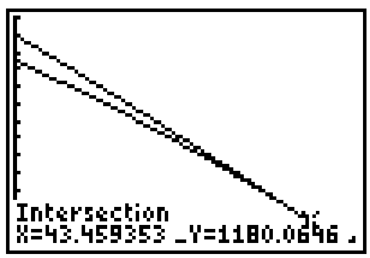
Step 4: Find the point of intersection for the two graphs.
Press **CALC** (2^{nd}, TRACE) and select **5:Intersect.** Press **ENTER** to select Y1 as the First Curve. Press **ENTER** again to select Y2 as the Second Curve. Press **ENTER** again to select the point where the cursor is located as the guess required by the calculator to perform this calculation.











QUESTION 1: What does this point of intersection mean in terms of the two loan options?

QUESTION 2: Find the value of Y1 when X = 0. Find the value of Y2 when X = 0. What do these two numbers represent?

QUESTION 3: If Tom decides to sell his car while he is still making payments on it, he'd like to owe as little as possible. Under which loan plan would Tom owe the least after 43 months?

SUGGESTED PRACTICE:

p 293 CKC (2 – 4)
p 294 E&A(12 – 21)

ANSWERS:
Question 1: Between 43 and 44 months, $1180 will still be owed with both loan options.
Question 2: When X = 0, Y1 = 12000 and Y2 = 10500. These are the initial amounts of the two loans.
Question 3: The first loan option has lower values for X > 43.

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Lesson 7.3 Solving Linear Systems with Matrices

Example 1

Matrices on the TI-83 may be used to solve systems of equations

P 303-4 Example 1:

Step 1: Write a system of equations to represent the problem.
Let X = the number of batches of sauce.
Let Y = the number of batches of juice.

4X + 8Y = 42
1X + 0.5Y = 6

Step 2: Rewrite the system of equations as a matrix equation.

$$\begin{matrix} 4 & 8 \\ 1 & 0.5 \end{matrix} \; \begin{matrix} X \\ Y \end{matrix} = \begin{matrix} 42 \\ 6 \end{matrix}$$

Check that when you multiply $\begin{matrix} 4 & 8 \\ 1 & 0.5 \end{matrix} \; \begin{matrix} X \\ Y \end{matrix}$ you get $\begin{matrix} 4X + 8Y \\ 1X + 0.5Y \end{matrix}$.

Step 3: Solve the matrix equation for $\begin{matrix} X \\ Y \end{matrix}$.

Multiply both sides of the equation by the inverse matrix for $\begin{matrix} 4 & 8 \\ 1 & 0.5 \end{matrix}$.

NOTE: Since multiplication of matrices is NOT commutative, you must multiply each side of the equation on the left by the inverse.

$$\begin{matrix} 4 & 8 \\ 1 & 0.5 \end{matrix}^{-1} \begin{matrix} 4 & 8 \\ 1 & 0.5 \end{matrix} \; \begin{matrix} X \\ Y \end{matrix} = \begin{matrix} 4 & 8 \\ 1 & 0.5 \end{matrix}^{-1} \begin{matrix} 42 \\ 6 \end{matrix}$$

QUESTION: What is $\begin{matrix} 4 & 8 \\ 1 & 0.5 \end{matrix}^{-1} \begin{matrix} 4 & 8 \\ 1 & 0.5 \end{matrix}$?

(Multiplying these inverses produces the identity $\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$ and

$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \; \begin{matrix} X \\ Y \end{matrix} = \begin{matrix} X \\ Y \end{matrix}$.)

Therefore,

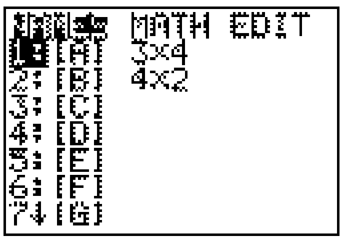
$$\begin{bmatrix} 4 & 8 \\ 1 & 0.5 \end{bmatrix}^{-1} \begin{bmatrix} 4 & 8 \\ 1 & 0.5 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 1 & 0.5 \end{bmatrix}^{-1} \begin{bmatrix} 42 \\ 6 \end{bmatrix} \text{ becomes}$$

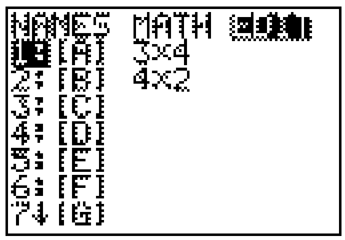
$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 1 & 0.5 \end{bmatrix}^{-1} \begin{bmatrix} 42 \\ 6 \end{bmatrix}$$

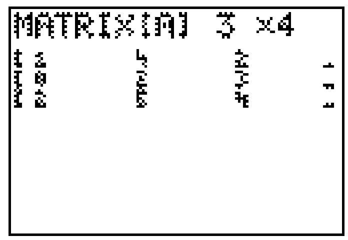
This multiplication step can be performed on the TI-83.

Step 4: Enter $\begin{bmatrix} 4 & 8 \\ 1 & 0.5 \end{bmatrix}$ as matrix [A] on the TI-83.

Press **MATRX** and press the right-arrow twice to highlight **EDIT.** Select **1:[A].**



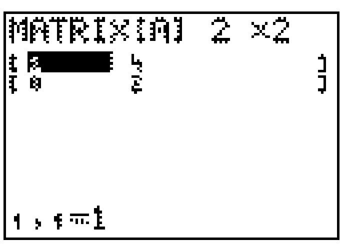


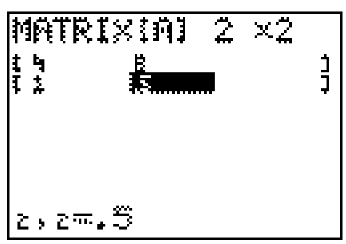


(Matrix [A] may look different on your screen depending on what you did last.)

The dimensions for $\begin{bmatrix} 4 & 8 \\ 1 & 0.5 \end{bmatrix}$ are 2 rows by 2 columns, so press **2, ENTER, 2,**

ENTER. Type in each element for $\begin{bmatrix} 4 & 8 \\ 1 & 0.5 \end{bmatrix}$ followed by **ENTER.**

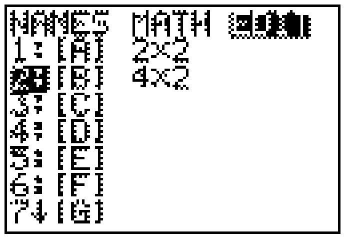


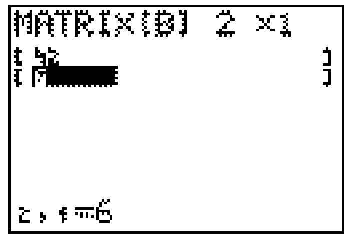


Press **QUIT** (2nd, MODE).

Step 5: Enter $\begin{bmatrix} 42 \\ 6 \end{bmatrix}$ as matrix [B] on the TI-83.

Press **MATRX** and press the right-arrow twice to highlight **EDIT.** Select **2:[B].** The dimensions for $\begin{matrix}42\\6\end{matrix}$ are 2 rows by 1 column, so press **2, ENTER, 1, ENTER.** Type each element of $\begin{matrix}42\\6\end{matrix}$ followed by **ENTER.**

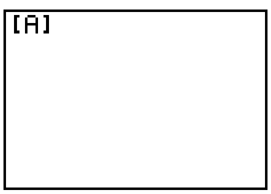


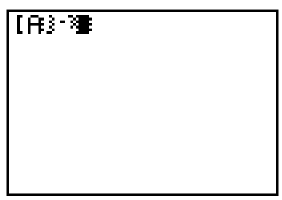


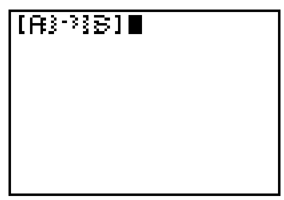
Press **QUIT** (2^{nd}, MODE).

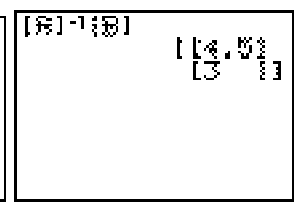
Step 6: Multiply $[A]^{-1}$ [B].

Press **MATRX** and with **NAMES** highlighted select **1:[A].** Press X^{-1}. Press **MATRX** and with **NAMES** highlighted select **2:[B].** Press **ENTER** to complete the multiplication.









The solution to the matrix equation

$$\begin{matrix}4 & 8\\1 & 0.5\end{matrix}\;\begin{matrix}X\\Y\end{matrix} = \begin{matrix}42\\6\end{matrix} \quad \text{is} \quad \begin{matrix}4.5\\3\end{matrix}$$

$$\begin{matrix}X\\Y\end{matrix} = \begin{matrix}4.5\\3\end{matrix}$$

or X = 4.5 and Y = 3. Therefore, Betsy needs to make 4.5 batches of sauce and 3 batches of juice.

NOTE: The majority of the work on solving systems of equations problems is in setting up the matrix equation and correctly interpreting the result from the calculator. It is not sufficient to type something into the calculator and write down an answer. You need to write down the matrix equation you are solving and a step that shows that you are multiplying each side on the left by the inverse of the coefficient matrix. The final step is to document the answer.

SUGGESTED PRACTICE:

p 305 Example 2
p 305 CKC (1 – 5)
p 306-7 E&A(1 – 26)

PPS
TI-83 Activities
Advanced Algebra 3-4

Lesson 10.3 Exploring Recursion

Example 3

Complete the EXPLORATION on p480.

This shows that the TI-83 is a recursive machine. The process used to generate a sequence by first entering a starting value and then repeatedly applying the same operation to the last answer is called **recursion.** A recursive formula for a sequence has two parts. The first part assigns a starting value. The second part (the recursion equation) defines t_n as a function of the previous term, t_{n-1}.

An **example of an arithmetic sequence defined recursively** is

$$t_1 = 5$$
$$t_n = t_{n-1} + 2$$

The first term is 5. To get the second term, add 2 to the first term.

$$t_2 = t_1 + 2$$
$$t_2 = 5 + 2$$
$$t_2 = 7$$

To get the third term, add 2 to the second term.

$$t_3 = t_2 + 2$$
$$t_3 = 7 + 2$$
$$t_3 = 9$$

The arithmetic sequence is 5, 7, 9, …

An **example of a geometric sequence defined recursively** is

$$t_1 = 5$$
$$t_n = 2(t_{n-1})$$

The first term is 5. To get the second term, multiply the first term by 2.

$$t_2 = 2(t_1)$$
$$t_2 = 2(5)$$
$$t_2 = 10$$

To get the third term, multiply the second term by 2.

$$t_3 = 2(t_2)$$
$$t_3 = 2(10)$$
$$t_3 = 20$$

The geometric sequence is 5, 10, 20, . . .

p 482 Example 3:

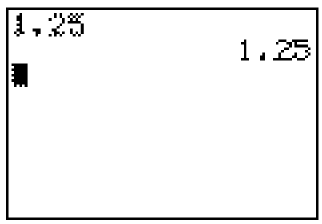
One medication used to treat high blood pressure is supplied in 1.25 mg tablets. Patients take one tablet at the same time every day. By the time a patient takes the next dose, only about 32% of the medication is left in the bloodstream. What happens to the level of medication in a patient's bloodstream over time?

Write a recursive formula to model the level of medication in the bloodstream after each dose.

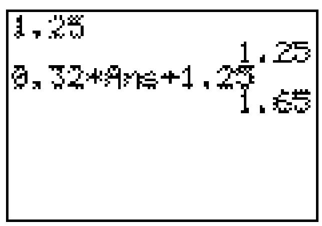
$$t_1 = 1.25$$
$$t_n = 0.32(t_{n-1}) + 1.25$$

Solution 1: We can solve this on the calculator using the process described in the EXPLORATION and the ANS key which produces the last answer calculated.

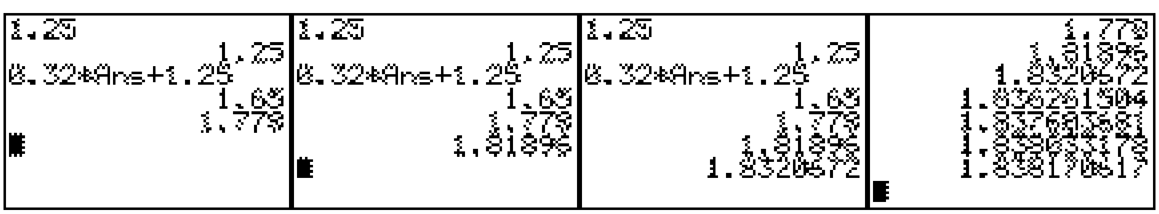
Type **1.25** and press **ENTER.**



Type **0.32, *, ANS** (2^{nd}, (-)), **+, 1.25** and press **ENTER.**



By pressing **ENTER** again the previous answer is multiplied by 0.32 and the result added to 1.25.

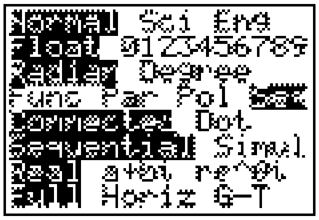


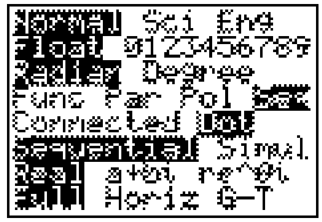
Continue to press **ENTER.** It appears that the amount of medication in the bloodstream levels off at about 1.84 mg.

The TI-83 will also graph the sequence points (n, t_n).

Solution 2:

Step 1: Press **MODE**. Press the down-arrow until FUNC is blinking. Then press the right-arrow until **Seq** is blinking and press **ENTER.**

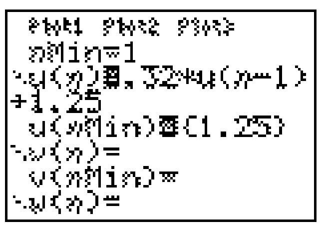




Press the down-arrow once and the right-arrow until **Dot** is blinking and press **ENTER.**

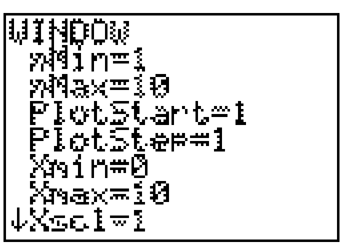
Seq is highlighted so the sequence mode is accessed, and Dot is highlighted because when graphing the points of a sequence as a function of the number of the term of the sequence, it is not appropriate to graph a continuous curve. The domain for a sequence function is the natural numbers. We should only see dots.

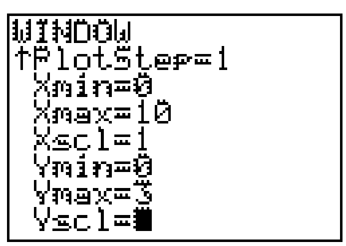
Step 2: Press **Y=** to enter the recursion equation. *N*Min = 1. To enter the recursion formula for $t_n = .32 * t_{n-1} + 1.25$ you need to use $u_n = .32 * u_{n-1} + 1.25$ because u, v, and w are the sequence variables on the TI-83. With u(*n*) highlighted, type .32*u(n-1)+1.25. u(*n*Min) = 1.25.



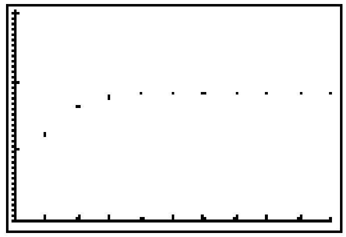
Typing Notes: To get the lower-case u, press 2nd, 7.
To get the n, press the (x, T, θ, n) key. When in sequence mode this key produces an n.

Step 3: Press **WINDOW** and enter the following values:





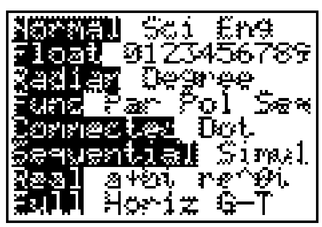
Step 4: Press **GRAPH.**



Press **TRACE** and the left- or right-arrow keys to read the values of the sequence.

After several days, the amount of medication in the bloodstream levels off at about 1.84 mg.

NOTE: Be sure to put your MODEs back to normal settings when you are done with sequence graphing.



SUGGESTED PRACTICE:

Use these techniques to check answers to p 484 E&A (10 – 21)
p 485 E&A(22)

PPS
TI-83 Activities
Advanced Algebra 3-4

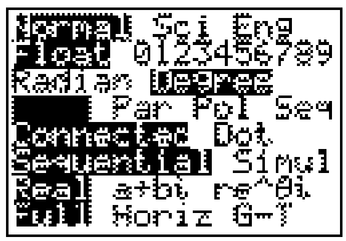
Lesson 15.1 Sine and Cosine on the Unit Circle

Example 1

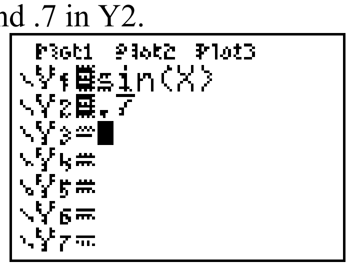
First complete the EXPLORATION on page 707 plotting values for $\sin\theta$ and $\cos\theta$ for $0^\circ \le \theta \le 360^\circ$ by hand.

p 708 Example 1 (a) :
Find all angles θ for $0^\circ \le \theta \le 360^\circ$ that satisfy the equation $\sin\theta = .7000$.

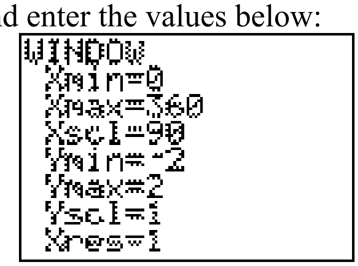
Step 1: Make sure the TI-83 is in Degree mode.
Press **MODE**. Press the down-arrow twice and the right-arrow once until Degree is blinking, and press **ENTER.**



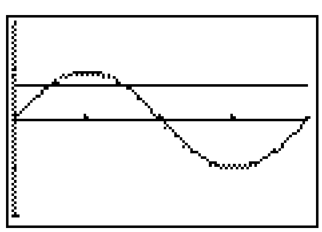
Step 2: Enter sin(X) in Y1 and .7 in Y2.



Step 3: Press **WINDOW** and enter the values below:

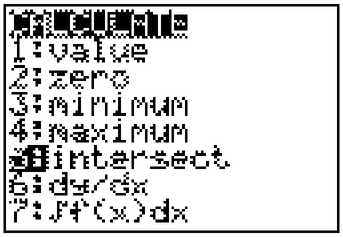


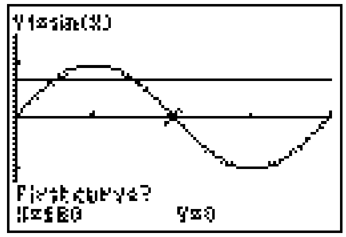
Step 4: Press **GRAPH.**



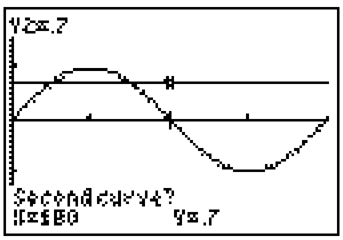
QUESTIION 1: How many solutions does sinθ =.7000 have for 0Y≤ θ ≤ 360Y? How can you tell?

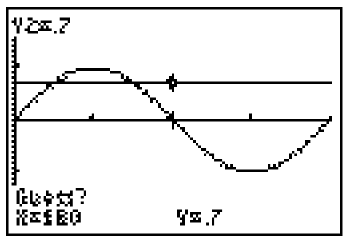
To find the points of intersection, press **CALC** (2^{nd}, TRACE) and select **5:intersect.**

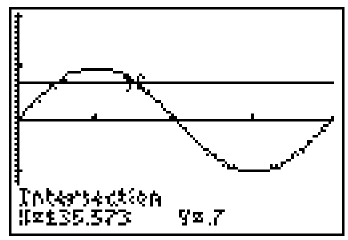




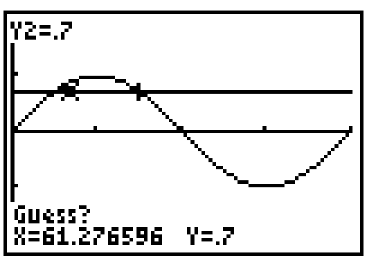
Press **ENTER** to select Y1=sin(X) as the first curve.
Press **ENTER** to select Y2 = .7 as the second curve.
Press **ENTER** to select the cursor position as the Guess.

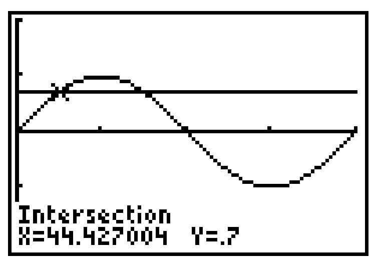






To find the other point of intersection, press **CALC** (2^{nd}, TRACE) and select **5:intersect** again. Press **ENTER** twice to select the curves, but move the cursor closer to the intersection point not found yet, and press **ENTER** again**.**





Therefore, θ 44Y and θ 136Y.

The solutions to cosθ = −0.4518 can be found in the same way.

SUGGESTED PRACTICE:

p 708 Example 1 (b)
p 710 CKC(1 – 6)
p 711 E&A (1 – 12)